BOOK III

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CHAPTER VI

EFFECTS OF AN ETHEREAL MEDIUM ON THE MOTIONS OF THE MOON

788. IN order to determine its effects in the hypothesis of its existence, let x, y, z be the co-ordinates of the moon referred to the centre of gravity of the earth, and x', y', z' those of the earth referred to the centre of the sun. The absolute velocity of the moon round the sun will be

$$\frac{\sqrt{(dx+dx')^2+(dy+dy')^2+(dz+dz')^2}}{dt}.$$

If K be a coefficient depending on the density of the ether, on the surface of the moon, and on her density; and if the resistance of the ether be assumed proportional to the square of the velocity, it will be

$$\frac{K\left\{(dx+dx')^2+(dy+dy')^2+(dz+dz')^2\right\}}{dt^2}.$$

In the same manner[1]

$$\frac{K'\left(dx'^2+dy'^2+dz'^2\right)}{dt^2}$$

is the resistance the earth experiences from the ether, K' being a coefficient for the earth similar to, but different from K. In the theory of the moon the earth is assumed to be at rest, therefore this resistance must be in a contrary direction from that acting on the moon, consequently the whole action of the ether in disturbing the moon will be the difference of these forces: so with regard to the action of the ether alone, (208) becomes

$$R=\frac{K'\left(dx'^2+dy'^2+dz'^2\right)}{dt^2}-\frac{K\left\{(dx+dx')^2+(dy+dy')^2+(dz+dz')^2\right\}}{dt^2}$$

and because the resistance is in the plane of the orbit, its component forces are parallel to the axes x and y only; hence

$$\frac{dR}{dx}=K'\frac{dx'}{dt^2}\cdot\sqrt{dx'^2+dy'^2+dz'^2}-K\frac{(dx+dx')}{dt^2}\cdot\sqrt{(dx+dx')^2+(dy+dy')^2+(dz+dz')^2}$$
$$\frac{dR}{dy}=K'\frac{dy'}{dt^2}\cdot\sqrt{dx'^2+dy'^2+dz'^2}-K\frac{(dy+dy')}{dt^2}\cdot\sqrt{(dx+dx')^2+(dy+dy')^2+(dz+dz')^2}$$

But in the theory of the moon

$$x=\frac{\cos v}{u},\;\; y=\frac{\sin v}{u},\;\; z=\frac{s}{u},\;\; x'=\frac{\cos v'}{u'},\;\; y'=\frac{\sin v'}{u'},$$

and if the ecliptic of 1750 be assumed as the fixed plane $z'=0: v'$ is the heliocentric longitude of the earth.

Let $\sqrt{dx'^2+dy'^2+dz'^2}$, the little arc described by the earth in the time dt be represented by $r'ds'$. This arc is to that described by the moon in her relative motion round the earth as $\frac{a'm}{a}$ to unity, consequently at least thirty times as great. If the eccentricity of the terrestrial orbit be omitted, $ds'=mdt$. If these quantities be substituted for the co-ordinates

$$\sqrt{(dx+dx')^2+(dy+dy')^2+(dz+dz')^2}=ma'dt-dx\,.\sin v'+dy\,.\cos v';$$

and if quantities depending on the arc $2v'$ be rejected,

$$\frac{dR}{dx}=\frac{(K-K')m^2}{u'^2}\cdot\sin v'-\frac{3Km}{2u'}\cdot\frac{dx}{dt}$$
$$\frac{dR}{dy}=\frac{(K-K')m^2}{u'^2}\cdot\cos v'-\frac{3Km}{2u'}\cdot\frac{dy}{dt}. \qquad \textbf{(249)}$$

But

$$d\frac{1}{a}=-2dR=-2dx\left(\frac{dR}{dx}\right)-2dy\left(\frac{dR}{dy}\right). \qquad \textbf{(250)}$$

and[2]

$$d\frac{1}{a}=-\frac{2(K-K')m^2}{u'^2}\cdot\{dx\cdot\sin v'-dy\,.\cos v'\}+\frac{3(K-K')m^2}{u'}\cdot\left\{\frac{dx^2+dy^2}{dt}\right\}. \qquad \textbf{(251)}$$

The different quantities contained in this equation must now be determined.

789. The distance of the moon from the earth is $Em=\frac{1}{u}$, that of the earth from the sun is $ES=\frac{1}{u'}$, and that of the moon from the sun is

$$mS=u'\sqrt{1+\frac{u'^2}{u^2}-2\frac{u'}{u}\cos(v-v')}$$

but $\frac{u'^2}{u^2}$ is a very small fraction that may be omitted; consequently, when the square root is extracted, the distance of the moon from the sun is

$$mS = u' - \frac{u'^2}{u}\cdot\cos(v - v').$$

If we assume the density of the ether to be proportional to a function of the distance from the sun, and represent that function by $f(u')$, with regard to the moon, it will be

$$f(u') - \frac{u'^2}{u}\cdot f'(u').\cos(v - v')$$

$f'(u')$ being the differential of $f(u')$ divided by du'. As K is a quantity depending on the density of the ether it is variable, hence it may be assumed that

$$K = H.f(u') - \frac{Hu'^2}{u}\cdot f'(u').\cos(v - v').$$

But as

$$x = \frac{\cos v}{u},\ \ y = \frac{\sin v}{u},\ \ u = \frac{1}{a}\left(1 + e\cos(cv - v\ )\right),$$

therefore

$$dx = -a^2(udv.\sin v + du.\cos v).\left(1 - 2e\cos(cv - v\ )\right),$$
$$dy = +a^2(udv\cdot\cos v - du\cdot\sin v)\cdot\left(1 - 2e\cos(cv - v\ )\right),$$

also

$$dt = dv\left(1 - 2e\cdot\cos(cv - v\ )\right).$$

790. By the substitution of these quantities in equation[3] (251) it will be found, after rejecting periodic quantities, and integrating, that[4]

$$\frac{1}{a} = -Hma^3\left\{\frac{3f(u')}{u'} - m\cdot f'(u')\right\}\cdot v + Hma^3\left\{\frac{6f(u')}{u'} - \frac{9}{2}m\cdot f'(u')\right\}\cdot e\sin(cv - v\ ),$$

which is the secular variation in the mean parallax of the moon in consequence of the resistance of the ether.

In order to abridge, let[5 6]

$$a = Hma^3\left\{\frac{3f(u')}{u'} - m\cdot f'(u')\right\},$$
$$x = Hma^3\left\{\frac{6f(u')}{u'} - \frac{9}{2}m\cdot f'(u')\right\},$$

then[7]

$$\frac{\bar{a}}{a} = -a\,v + x\,.\,e\sin\left(cv - v\right).$$

The value of $\frac{1}{a}$ in equation (225) will be augmented by $a\,v$, therefore a will be diminished by $a\,v$. Since

$$d\,\frac{1}{a} = -2\mathrm{d}R\,,$$

therefore

$$\mathrm{d}R = \frac{a}{2\bar{a}}\,dv - \frac{x}{2\bar{a}}\,dv\,.\,e\,.\cos\left(cv - v\right).$$

Consequently, when periodic quantities are omitted, $z = -3\int a\,dv\,.\,\mathrm{d}R$ gives

$$z = -\frac{3a}{4\bar{a}}\,av^2$$

or, omitting the action of the sun,

$$z = -\tfrac{3}{4}\,a\,v^2.$$

Thus the mean motion is affected by a secular variation from the resistance of the ethereal medium; but it may easily be shown, from the value of R in article 788, that this medium has no effect whatever on the motion of the lunar nodes or perigee. However, in consequence of that action the second of equations (224), which is the coefficient of $\sin\left(cv - v\right)$, ought to be augmented by $x\,.\,e$; hence, rejecting c^2, dv, and making $c = 1$ it gives[8]

$$\frac{x\,.\,edv}{\bar{a}} = 2\,.\,d\,\frac{e}{a},$$

or

$$\frac{e}{a} = \text{constant}\,\left(1 + \tfrac{1}{2}xv\right);$$

but as $\frac{1}{a}$ must be augmented by $a\,v$, if the square of v be omitted,

$$e = \text{constant}\,\left(1 - \left(a - \tfrac{1}{2}x\right)v\right).$$

Thus the eccentricity of the lunar orbit is affected by a secular inequality from the resistance of ether, but it is insensible when compared with the corresponding inequality in the mean motion.

It appears then that the mean motion of the moon is subject to a secular variation in consequence of the resistance of ether, which neither affects the motion of the perigee nor the position of the orbit; and, as the secular inequalities of the moon deduced theoretically from the variation of the eccentricity of the earth's orbit are perfectly confirmed by the concurrence of ancient and modern observations, they cannot be ascribed to the resistance of an ethereal medium.

791. The action of the ether on the motions of the earth may be found by the preceding formulae to be

$$\frac{dR}{dx} = +K'm^2a'^2 \cdot \sin v'$$
$$\frac{dR}{dy} = -K'm^2a'^2 \cdot \cos v';$$

when the eccentricity of the earth's orbit is omitted, so that

$$u' = \frac{1}{a'}.$$

Consequently the general equation (250) gives

$$\text{d}R = -K'.a'^3.m^3.dt, \text{ and therefore}$$
$$d\,v = -\frac{3a'}{m'} \cdot \iint dt \,.\, \text{d}R = \tfrac{3}{2}\frac{K'.a'^4 m^4.t^2}{m'},$$

m' being the mass of the sun.

If $f(u')$ be a function of the distance of the earth from the moon, then must

$$K' = H' \cdot f(u'),$$

H' being a constant quantity depending on the mass and surface of the earth. Whence it may be found by the same method with that employed, that the resistance of ether in the mean motion of the earth would be

$$z = \tfrac{3}{2}\frac{H'a'^4m^4t^2 \cdot f(u')}{m'}.$$

Whence it appears that the acceleration in the mean motion of the moon is to that in the mean motion of the earth as unity to

$$\frac{2H' \cdot m \cdot f(u')}{H\left\{3f(u') - \frac{m}{a'}f'(u')\right\}},$$

or as unity to

$$\tfrac{2}{3}m.\frac{H'}{H}, \text{ if } -\frac{m}{a'}f'(u')$$

be omitted, and because

$$\frac{m'a^3}{a^3} = m^2 .$$

Now H' and H depend on the masses and surfaces of the earth and moon; and as the resistance is directly as the surface, and inversely as the mass, therefore

$$H = \frac{\text{surface}}{\text{mass}} .$$

But by article 652, if the radius of the earth be unity, the moon's true diameter =

$$\frac{\tfrac{1}{2}\text{ moon's apparent diameter}}{\text{moon's horizontal parallax}};$$

hence surface of moon[9] =

$$\frac{\left(\tfrac{1}{2}\text{ apparent diameter}\right)^2}{(\text{lunar parallax})^2}$$

and

$$H = \frac{\left\{\tfrac{1}{2}\text{ apparent diameter of moon}\right\}^2}{\text{mass of moon }\{\text{lunar parallax}\}^2} .$$

But as the terrestrial radius is assumed = 1, the earth's surface is unity; so

$$H' = \frac{1}{\text{mass of earth}};$$

hence

$$\frac{H'}{H} = \frac{\text{mass of moon}}{\text{mass of earth}}\cdot\frac{\text{square horizontal parallax of moon}}{\text{square of }\tfrac{1}{2}\text{ moon's apparent diameter}} .$$

From observation half the moon's apparent diameter is $943''.164$, her horizontal parallax is $3{,}454.16$, and her mass is $\frac{1}{75}$ of that of the earth, so $\frac{H'}{H} = 0.17883$; and as $m = \frac{1}{13.3}$, it follows that the acceleration in the mean motion of the earth from the resistance of ether is equal to the corresponding acceleration in the mean motion of the moon multiplied by 0.008942, or about a hundred times less than the acceleration of the moon from the resistance of ether. No such acceleration has been detected in the earth's motion, nor could it be expected, since it is insensible with regard to the moon.

In the preceding investigation, the resistance was assumed to be as the square of the velocity, but Mr. Lubbock[10] has obtained general formulae, which will give the variations in the elements, whatever the law of this resistance may be.

792. Although we have no reason to conclude that the sun is surrounded by ether, from any effects that can be ascribed to it in the motions of the moon and planets, the question of the existence of such a fluid has lately derived additional interest from the retardation that has been observed in the returns of Encke's comet[11] at each revolution, which it is difficult to account for by any other supposition than this existence of such a medium.

Mr. Enke has proved that this retardation does not arise from the disturbing action of the planets. But on computing the effects of the resistance of an ether diffused through space, he found that the diminution in the periodic time, and on the eccentricity arising from the ether, supposing it to exist, corresponds exactly with observation. This coincidence is very remarkable, because ignorance of the nature of the medium in question imposes the necessity of forming an hypothesis of the law of its resistance. Future returns of this comet will furnish the best proof of the existence of an ether, which, by the computation of Mazotti, must be 360,000 millions of times more rare than atmospheric air, in order to produce the observed retardation. The existence of an ethereal medium, if established, would not only be highly important in astronomy, but also from the confirmation it would afford of the undulating theory of light; among whose chief supporters we have to number Huygens,[12] Descartes,[13] Hooke,[14] Euler,[15] and, in later times, the illustrious names of Young[16] and Fresnel,[17] who have applied it with singular success and ingenuity to the explanation of those classes of phenomena which present the greatest difficulties to the corpuscular doctrine.

793. Laplace[18] employs the same analysis to determine the effects that the resistance of light has on the motions of the bodies of the solar system, whether considered as propagated by the undulations of a very rare medium as ether, or emanating from the sun. He finds that it has no effect whatever on the motion of the perigee, either of the sun or moon; that its action on the mean motions of the earth and moon is quite insensible; but that the action of light, on the mean motion of the moon, in the corpuscular hypothesis, is to that in the undulating system as -1 to 0.01345.

794. If gravitation be produced by the impulse of a fluid towards the centre of the attracting body, the same analysis will give the secular equation due to the successive transmission of the attractive force. The result is, that if g be the attraction of any body as the earth; G the ratio of the velocity of the fluid which causes gravitation to that of the moon, at her mean distance, and t any finite time, the secular equation of the mean motion of the moon from the transmission of the attractive force is $\tfrac{3}{2}\frac{gt^2}{aG}$.

The gravity of a body moving in its orbit is equal to its centrifugal force; and the latter is equal to the nature of the velocity divided by the radius vector; and as the square of the moon's velocity is[19] $a(27.32166)^2$ its centrifugal force is $(27.32166)^2$, whence[20]

$$g = a(27.32166)^2;$$

and the secular equation becomes

$$\tfrac{3}{2}\left(\frac{(27.32166)^2}{G}\right)\cdot t^2 .$$

Since G is the ratio of the velocity of the fluid in question to the velocity of the moon

$$G = \frac{\text{velocity of the fluid}}{a(27.32155)};$$

hence the velocity of the fluid is $(27.32166)\,aG$.

If

$$L = \frac{\text{velocity of the fluid}}{\text{velocity of light}},$$

then the velocity of the gravitating fluid is equal to L [times the] velocity of light; whence L [times the] velocity of light = (27.32166)aG; but by Bradley's theory,[21] the velocity of light is[22]

$$\frac{(365.25)\,a'}{\tan 20''.25},$$

a' being the mean distance of the earth from the sun; whence

$$L\cdot\frac{(365.25)\,a'}{\tan 20''.25} = (27.32166)\,aG,$$

$$G = \frac{L(365.25)\,a'}{(27.32166)\,a\cdot\tan 20''.25}.$$

And the secular equation of the moon from the successive transmission of gravity becomes[23]

$$\tfrac{3}{2}\frac{(27.32166)^3}{L(365.25)}\cdot\frac{a}{a'}\cdot t^2\cdot\tan 20''.25.$$

Now, if the acceleration in the moon's mean motion arises from the successive transmission of gravity, and not from the secular variation in the earth's eccentricity, the preceding expression would be equal to $10''.1816213$, the acceleration in 100 Julian years. Therefore, making $t = 100$,

$$L = \tfrac{3}{2}\frac{a}{a'}\frac{(27.32166)^3}{365.25}\cdot\frac{10{,}000\tan 20''.25}{10''.1816213};$$

but

$$\frac{a}{a'} = \frac{1}{400};$$

whence[24]

$$L = 50,464,700;$$

thus the velocity with which gravity is transmitted must be more than fifty[25] million times greater than the velocity of light:[26] hence we must suppose the velocity of the moon to be many a hundred million times greater than that of light to preserve her from being drawn to the earth, if her acceleration be owing to the successive transmission of gravity. The action of gravity may therefore be regarded as instantaneous.

795. These investigations are general, though they have only been applied to the earth and moon; and, as the influence of the ethereal media and of the transmission of gravity on the moon is quite insensible, though greater than on the earth, it may be concluded, that they have no sensible effect on the motions of the solar system; but as they do not affect the motions of the lunar perigee and the perihelia of the earth and planets at all, these motions afford a more conclusive proof of the law of gravitation, than any other circumstance in the system of the world. The length of the day is proved to be constant by the secular equation of the moon. For if the day were longer now than in the time of Hipparchus[27] by the 0.00324th of a second, the century would be $118''.341$ longer than at that period. In this interval, the moon would describe an arc of $173''.2$, and her actual mean secular motion would appear to be augmented by that quantity; so that her acceleration, which is $10''.206$ for the first century, beginning from 1801, would be increased by $4''.377$; but observations do not admit of so great an increase. It is therefore certain, that the length of the day has not varied the 0.00324th of a second since the time of Hipparchus.

796. It is evident then, that the lunar motions can be attributed to no other cause than the gravitation of matter: of which the concurring proofs are the motion of the lunar perigee and nodes; the mass of the moon; the magnitude and compression of the earth; the parallax of the sun and moon, and consequently the magnitude of the system; the ratio of the sun's action to that of the moon, and the various secular and periodic inequalities in the moon's motions, every one of which is determined by analysis on the hypothesis of matter attracting inversely as the square of the distance; and the results thus obtained, corroborated by observation, leave not a doubt that the whole obey the law of gravitation. Thus the moon is, of all the heavenly bodies, the best adapted to establish the universal influence of this law of nature; and, from the intricacy of her motions, we may form some idea of the powers of analysis, that marvelous instrument, by the aid of which so complicated a theory has been unraveled.

797. Before we leave the subject, it may be interesting to show that the differential equations of the lunar co-ordinates, given in (207), may be derived from Newton's theory.

If the inclination of the lunar orbit be omitted, the whole force which disturbs the moon may be resolved into two; one perpendicular to the radius vector, and another, according to the radius vector, and directed towards the centre of the earth. Now, $\frac{1}{r}\left(\frac{dR}{dv}\right)$ is the first of these forces, and $-\left(\frac{dR}{dr}\right)$ is the other. The force $\frac{1}{r}\left(\frac{dR}{dv}\right)$, multiplied by dt, gives the increment of the velocity of the moon perpendicular to the radius during the instant dt; and when multiplied by $\tfrac{1}{2}rdt$, it becomes $\tfrac{1}{2}\left(\frac{dR}{dv}\right)dt$ = the increment of the area described by the radius vector in the time dt. It must therefore be equal to $\tfrac{1}{2}\cdot\frac{d\,.r^2dv}{dt}$; hence

$$\frac{d\,.r^2dv}{dt}=\left(\frac{dR}{dv}\right)dt\,.$$

If this equation be multiplied by $\frac{r^2dv}{dt}$ and integrated, the result will be

$$\left(r^2dv\right)^2=h^2dt^2\left(1+\frac{2}{h^2}\int\left(\frac{dR}{dv}\right)r^2dv\right);$$

and as $r=\frac{1}{u}$, it becomes

$$dt=\frac{dv}{hu^2\sqrt{1+\frac{2}{h^2}\int\left(\frac{dR}{dv}\right)\frac{dv}{u^2}}},$$

which is the first of equations (207).

Again, if ds be the element of the curve described by the moon, $\frac{ds^2}{dt^2}$ will be the square of her velocity; and, substituting the preceding value of dt, the square of the moon's velocity will be

$$h^2u^4\cdot\frac{ds^2}{dv^2}\cdot\left\{1+\frac{2}{h^2}\int\left(\frac{dR}{dv}\right)\cdot\frac{dv}{u^2}\right\}.$$

If $r`$ be the oscillating radius of the orbit, the expression of the radius of curvature, in article 83, will give, when substitution is made for x, y, z, in supposing dv constant,

$$\frac{1}{r`}=dv^3\,\frac{\left(\frac{d^2u}{dv^2}+u\right)}{u^3ds^3}.$$

Hence the square of the moon's velocity, divided by the radius of curvature, is

$$u \cdot \frac{dv}{ds} \cdot h^2 \left\{ \frac{d^2u}{dv^2} + u \right\} \cdot \left\{ 1 + \frac{2}{h^2} \int \left(\frac{dR}{dv} \right) \frac{dv}{u^2} \right\}. \quad \textbf{(252)}$$

By the theorems of Huygens, this expression must be equal to the lunar force resolved in the radius of curvature, and directed towards the centre of curvature. Now, if the force $-\left(\frac{dR}{dr}\right)$ be resolved into two, one parallel to the element of the curve, and the other directed to the centre of curvature, the latter will be $u\left(\frac{dR}{du}\right)\cdot\frac{dv}{ds}$. Also the force $\frac{1}{r}\left(\frac{dR}{dv}\right)$, resolved according to the radius of curvature, will be $-\frac{du}{uds}\left(\frac{dR}{dv}\right)$. The sum of these two forces directed towards the centre of curvature is

$$u \cdot \frac{dv}{ds}\left(\frac{dR}{du}\right) - \frac{du}{uds}\left(\frac{dR}{dv}\right).$$

If the square of this expression be made equal to that of (252), then

$$0 = \left(\frac{d^2u}{dv^2} + u\right)\left\{1 + \frac{2}{h^2}\int\left(\frac{dR}{dv}\right)\frac{dv}{u^2}\right\} - \frac{1}{h^2}\left(\frac{dR}{du}\right) + \frac{du}{h^2u^2dv}\left(\frac{dR}{dv}\right);$$

which is the same with the second of equations (202), when the inclination of the orbit is omitted.

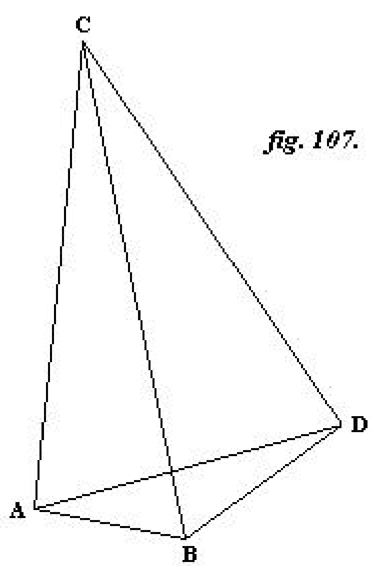

fig. 107.

The equation in latitude is not so easily found as the other two; but the method followed by Newton was to resolve the action of the sun on the moon into two, one in the direction of the radius vector of the lunar orbit, the other parallel to a line joining the centres of the sun and earth. The difference between the last force and the action of the sun on the earth, he saw to be the only force that could change the position of the lunar orbit, since it is not in that plane. He determined the effect of this force, by supposing AB, fig. 107, to be the arc described by the moon in an instant; then ABC is the plane of the orbit during that time; in the next instant, the difference of the two forces causes the moon to describe the small arc BD in a different plane; then if BD represent the difference of the forces, and if AB be the velocity of the moon in the first instant, the diagonal BD will be the direction of the velocity in the second instant; and ACD will be the position of the orbit. Newton deduced the horary and mean motion of the nodes, their principal variation, and the inequalities in latitude, from these considerations. Laplace considered the theory of the moon as the most profound and ingenious part of the Principia.

Notes

[1] The closing parenthesis in the denominator is missing in the 1st edition.

[2] $(K - K')$ *reads* K in equation (251) in the 1st edition (published erratum).

[3] This *reads* "equation (241)" in the 1st edition (published erratum).

[4] The multipliers a^3 *read* a^2 in the 1st edition (published erratum).

[5] The multipliers a^3 *read* a^2 in the 1st edition (published erratum).

[6] The term $\frac{3f(u')}{u'}$ *reads* $\frac{3f(n')}{u'}$ in the 1st edition.

[7] $\frac{\overline{a}}{a}$ *reads* $\frac{1}{a}$ in the 1st edition (published erratum).

[8] The left hand side *reads* $x\ .edv$ in the 1st edition (published erratum).

[9] The factor $\frac{1}{2}$ in the numerator is omitted in the 1st edition (published erratum).

[10] Lubbock, John William, Sir, 1803-1865, *On the theory of the moon, and on the perturbations of the planets*, London : C. Knight, 1833.

[11] Encke, Johann Franz, 1791-1865, *On Encke's comet, Encke's dissertation contained in no. CCX and CCXI of the Astonomische Nachrichten ; translated from German by G. B. Airy*, Cambridge : Printed by. J. Smith, 1832.

[12] See note 12, *Bk. II, Chap. II.*

[13] Descartes, René, 1596-1650, philosopher and mathematician, born in La Haye, France. His principle philosophic work is his *Meditationes de prima philosophia* (1641, Meditations on First Philosophy). His *Discours de la méthode pour bien conduire sa raison, et chercher la vérité dans les sciences* (1637, Discourse on the Method for Rightly Conducting One's Reason and Searching for Truth in the Sciences) contains an appendix in which he establishes the foundational principles of analytic geometry. Descartes also formulated a mechanical "vortex" model of planetary motion in which the earth and other planets in contact with air whirled about the sun.

[14] See note 14, *Bk. II, Chap. I.*

[15] See note 6, *Bk. I, Chap. II.*

[16] See note 35, *Preliminary Dissertation.*

[17] See note 51, *Preliminary Dissertation.*

[18] See note 4, *Introduction.*

[19] This *reads* $a^2(27.32166)^2$ in the 1st edition (published erratum).

[20] This *reads* $g = (27.32166)^2$ in the 1st edition (published erratum).

[21] See note 38, *Preliminary Dissertation.*

[22] The numerator *reads* $(365.25)a$ in the 1st edition (published erratum).

[23] L is omitted from the denominator in the 1st edition (published erratum).

[24] This *reads* L = 42,145,000 in the 1st edition (published erratum).

[25] This *reads* "forty-two" in the 1st edition. This is the final published erratum in the 1st edition.

[26] The phrase "the velocity of light" is repeated twice in the 1st edition.

[27] See note 32, *Preliminary Dissertation.*