BOOK III - LUNAR THEORY

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FOREWORD[1]

Rotation of the Moon

THE periods of rotation of the moon and the other satellites are equal to the times of their revolutions, consequently these bodies always turn the same face to their primaries. However, as the mean motion of the moon is subject to a secular inequality, which will ultimately amount to many circumferences if the rotation of the moon were perfectly uniform and not affected by the same inequalities, it would cease exactly to counterbalance the motion of revolution; and the moon, in the course of ages, would successively and gradually discover every point of her surface to the earth. But Laplace showed that this never can happen; for the rotation of the moon, though it does not partake of the periodic inequalities of her revolution, is affected by the same secular variations, so that her motions of rotation and revolution round the earth will always balance each other, and remain equal. This circumstance can only be accounted for by the form of the moon herself. She has three principal axes at right angles to each other. The first of these, the polar axis, is supposed to be the least, because, according to theory, the moon is flattened at her poles from her centrifugal force. The other two axes are in the plane of her equator, and of these the one which is directed towards the earth is the greatest. The attraction of the earth, as if it had drawn out that part of the moon's equator, constantly brings the greatest axis, and consequently the same hemisphere, towards us, which makes her rotation participate in the secular variations of her mean motion of revolution. Even if the angular velocities of rotation and revolution had not been nicely balanced in the beginning of the moon's motion, the attraction of the earth would have recalled the greatest axis to the direction of the line joining the centres of the moon and earth; so that it would have vibrated on each side of that line in the same manner as a pendulum oscillates on each side of the vertical from the influence of gravitation. No such libration[2] is perceptible; but another movement very similar, which was theoretically proved by Newton, has been detected by some observers. This movement, which is called *physical libration*, depends upon the fact, that as the moon does not move with equal velocity in all parts of her orbit, her elongated axis is not always directed exactly to the earth, but oscillates perpetually on each side of its mean place.

Besides these almost imperceptible movements, the moon is subject to very important librations which cause us to see, from time to time, slightly round the edge of her globe. These, which are called *libration in latitude* and *libration in longitude*, are due to two different causes, namely, the inclination of the moon's axis of rotation to the plane of her orbit, and the balance between her motions of rotation and revolution. The moon's equator-plane is inclined $1^\circ 30' 10''.8$ to the ecliptic, and in consequence of this inclination her northern and southern poles lean alternately in a slight degree to and from the earth, so that the middle of her visible disc lies at one time $6^\circ 39'$ north of the equator, and at another time as far south. This is the

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[1] The material in this and the forewords to Books I, II and IV is extracted from the 10th and last edition of Mary Somerville's *On the Connexion of the Physical Sciences*, (corrected and revised by Arabella B. Buckley), p. 4-106, London : John Murray, 1877.

[2] *libration.* A balancing movement to and fro (Somerville's note).

libration in latitude. The second libration, or the *libration in longitude*, is caused by the fact, that while the moon rotates with perfect uniformity on her axis, she does not move with equal velocity in all parts of her orbit on account of its eccentricity. When in perigee her movement is accelerated, and her revolution outstrips her rotation; when in apogee her movement is retarded, and she has turned round on her axis more than she has advanced in her orbit. The consequence of this is, that a meridian drawn across the centre of her true disc lies sometimes to the west and sometimes to the east of her visible disk, according as her rotation or revolution are in advance, and we see more of the eastern or western halves of her surface. As the eccentricity of the moon's orbit is variable, the amount of her libration in longitude is also variable. There is also another libration of very slight importance, called *diurnal libration*, which depends upon the fact that we are on the surface of a rotating globe, which causes the moon to appear to oscillate about her radius vector. This libration never exceeds $1^\circ 1' 28''.8$.

For the same reason that one hemisphere of the moon must remain eternally concealed from us, the earth, which must be so splendid an object to one lunar hemisphere, will be for ever veiled from the other. On account of these circumstances, the remoter hemisphere of the moon has its day a fortnight long, and a night of the same duration, not even enlightened by a moon, while the favoured side is illuminated by the reflection of the earth during its long night. A planet exhibiting a surface thirteen time's larger than that of the moon, with all the varieties of clouds, land, and water coming successively into view, would be a splendid object if seen from the moon. The great height of the lunar mountains probably has a considerable influence on the phenomena of her motion, the more so as her compression is small and her mass considerable. In the curve passing through the poles, and that diameter of the moon which always points to the earth, nature has furnished a permanent meridian to which the different spots on her surface have been referred, and their positions are determined with as much accuracy as those of many of the most remarkable places on the surface of our globe. The surface of the moon has been of late years most extensively studied, and some of the results are very remarkable. Not only is that face of the planet which is turned towards us covered with large extinct volcanic craters, some of these having a diameter of from 40 to 50 miles, and a depth of 10,000 feet; but large circular formations, with no cones in the centre, and measuring from 100 to 300 miles across, also occur. Their origin is not easy to explain, though Professor Dana[3] has suggested that they may once have been boiling lakes of lava, like the pit-crater of Kilauea in the Hawaiian Islands. Besides these evidently volcanic productions, there are the peaks and mountain ranges which have been generally supposed to be caused by the shrinking of the moon's surface, but which Messrs. Nasmyth and Carpenter[4] suggest may have been also formed by successive flows of lava oozing out from the crust and solidifying in slopes which would grow gradually steeper and steeper. One of the most remarkable appearances, however, is that of systems of streaks of light and shade which radiate from the borders of some of the largest ring mountains, and spread for hundreds of miles, having the appearance of glass or ice which has been starred by a blow. It is now generally believed that these streaks are cracks caused by the rending of the solid crust of the moon, and afterwards filled with molten matter from beneath.

[3] Dana, James Dwight, (1813-1895), Geologist, born in Utica, N.Y., USA. Dana was an editor of the American Journal of Science (1846-95). He wrote a text on Hawaiian volcanoes *(Characteristics of volcanoes*, New York, Dodd, Mead and company, 1891) and several books on geology including *Manual of Geology* (1862) and a highly successful book on mineralogy *A System of Mineralogy* (1837). Dana was a personal acquaintance of Mary Somerville, and corresponded with botanist Asa Gray, naturalist Louis Agassiz, and Charles Darwin.

[4] Nasmyth and Carpenter, *The Moon*, 1874. (Somerville's note.)

Lunar Perturbations

Several circumstances concur to render the moon's motions the most interesting, and at the same time the most difficult to investigate, of all the bodies of our system. In the solar system, planet troubles planet; but in the lunar theory, the sun is the great disturbing cause, his vast distance being compensated by his enormous magnitude, so that the motions of the moon are more irregular than most of the planets; and, on account of the great ellipticity of her orbit, and the size of the sun, the approximations of her motions are tedious and difficult, beyond what those unaccustomed to such investigations could imagine. The average distance of the moon from the centre of the earth is only 238,818 miles,[5] so that her motion among the stars is perceptible in a few hours. She moves in an orbit whose eccentricity is about 12,985 miles, and completes a circuit of the heavens in $27^d\ 7^h\ 43^m\ 11^s.5$, although to the fact that the earth is also moving onward, the exact time from one new moon to another is $29^d\ 12^h\ 44^m\ 2^s.87$. The moon is about four hundred times nearer to the earth than the sun. The proximity of the moon to the earth keeps them together; for so great is the attraction of the sun, that, if the moon were farther from the earth, she would leave it altogether, and would revolve as an independent planet about the sun.

The disturbing action of the sun on the moon is equivalent to three forces. The first, or radial force, acting in the direction of the line joining the moon and earth, increases or diminishes her gravity to the earth. The second, or tangential force acting in the direction of a tangent to her orbit, disturbs her motion in longitude. And the third, or perpendicular force, acting perpendicularly to the plane of her orbit, disturbs her motion in latitude; that is, it brings her nearer to, or removes her farther from, the plane of the ecliptic than he would otherwise be. The periodic perturbations in the moon, arising from these forces, are perfectly similar to the periodic perturbations of the planets. But they are much greater and more numerous; because the sun is so large, that many inequalities which are quite insensible in the motions of the planets are of great magnitude in those of the moon. Among the innumerable periodic inequalities to which the moon's motion in longitude is liable, the most remarkable are, the Equation of the Centre (see Article 382, *Bk. II*, *Chap. IV*), which is the difference between the moon's mean and true longitude, the Evection, the Variation, and the Annual Equation. The disturbing force which acts in the line, joining the moon and earth produces the Evection:[6] it diminishes the eccentricity of the lunar orbit in conjunction and opposition, thereby making it more circular, and augments it in quadrature, which consequently renders it more elliptical. The period of this inequality is less than thirty-two days. Were the increase and diminution always the same, the Evection would only depend upon the distance of the moon from the sun; but its absolute value also varies with her distance from the perigee,[7] of her orbit. Ancient astronomers who observed the moon solely with a view to the prediction of eclipses, which can only happen in conjunction and opposition, where no eccentricity is diminished by the Evection, assigned too small a value to the ellipticity

[5] Modern value is 238,866 miles (384,400 km).

[6] *Evection.* The evection is produced by the action of the radial force acting along the line joining the earth and the moon. It sometimes increases and sometimes diminishes the earth's attraction o the moon. It produces a corresponding temporary change in the eccentricity, which varies with the position of the major axis of the lunar orbit in respect of the line joining the centres of the earth and the sun. (Somerville's note.)

[7] *Perigee.* A Greek word signifying round the earth. The perigee of the lunar orbit is the point where the moon is nearest to the earth. It corresponds to the perihelion of a planet (see art. 316, *Bk. II, Chap. II*). Sometimes the word is used to denote the point where the sun is nearest the earth. (Somerville's note.)

of her orbit. The Evection was discovered by Ptolemy,[8] from observation, about AD 140. The Variation produced by the tangential disturbing force, which is at its maximum when the moon is 45° distant from the sun, vanishes when that distance amounts to a quadrant, and also when the moon is in conjunction and opposition; consequently, that inequality never could have been discovered from the eclipses: its period is half a lunar month.[9] The Annual Equation depends upon the sun's distance from the earth: it arises from the moon's motion being accelerated when that of the earth is retarded, and *vice versâ*–for, when the earth is in its perihelion, the lunar orbit is enlarged by the action of the sun; therefore, the moon requires more time to perform her revolution. But, as the earth approaches its aphelion, the moon's orbit contracts, and less time is necessary to accomplish her motion–its period, consequently, depends upon the time of the year. In eclipses the Annual Equation combines with the Equation of the Centre of the terrestrial orbit, so that ancient astronomers imagined the earth's orbit to have a greater eccentricity than modern astronomers assign to it.

The planets disturb the motion of the moon both directly and indirectly; their action on the earth alters its relative position with regard to the sun and moon, and occasions inequalities in the moon's motion, which are more considerable than those arising from their direct action; for the same reason the moon, by disturbing the earth, indirectly disturbs her own motion. Neither the eccentricity of the lunar orbit, nor its mean inclination to the plane of the ecliptic, have experienced any changes from secular inequalities; for, although the mean action of the sun on the moon depends upon the inclination of the lunar orbit to the ecliptic, and the position of the ecliptic is subject to a secular inequality, yet analysis shows that it does not occasion a secular variation in the inclination of the lunar orbit, because the action of the sun constantly brings the moon's orbit to the same inclination to the ecliptic. The mean motion, the nodes, and the perigee, however, are subject to very remarkable variations.

From the eclipse observed at Babylon, on March 19, seven and twenty-one years before the Christian era, the place of the moon is known from that of the sun at the instant of opposition, whence her mean longitude may be found. But the comparison of this mean longitude with another mean longitude, computed back for the instant of the eclipse from modern observations, shows that the moon performs her revolution round the earth more rapidly and in a shorter time now than she did formerly, and that the acceleration in her mean motion has been increasing from age to age as the square of the time.[10] All ancient and intermediate eclipses confirm this result. As the mean motions of the planets have no secular inequalities, this seemed to be an unaccountable anomaly. It was at one time attributed to the resistance of an ethereal medium pervading space, and at another to the successive transmission of the gravitating force. But, as Laplace proved that neither of these causes, even if they exist, have any influence on the motions of the lunar perigee or nodes, they could not affect the mean motion; a variation in the mean motion from such causes being inseparably connected with variations in the motions of the perigee and nodes. That great mathematician, in studying the theory of Jupiter's satellites, perceived that the secular variation in the elements of Jupiter's orbit, from the action of the

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[8] See note 15, *Preliminary Dissertation*.

[9] *Variation*. The lunar perturbation called the variation is the alternate acceleration and retardation of the moon in longitude, from the action of the tangential force. She is accelerated when approaching the two points located 90 degrees from the perigee and apogee (called the syzygies), and retarded when returning from the syzygies to the perigee or apogee. (Somerville's note.)

[10] *Square of the time*. If the times increase at the rate of 1, 2, 3, 4, &c., years or hundreds of years, the squares of the times will be 1, 4, 9, 16, &c., years or hundreds of years (Somerville's note).

planets, occasions corresponding changes in the motions of the satellites, which led him to suspect that the acceleration in the mean motion of the moon might be connected with the secular variation in the eccentricity of the terrestrial orbit. Professor Adams[11] has now shown that the whole amount cannot be explained by secular variation, and it is probable that a part of it is only apparent, the real cause being a retardation in the earth's motion of rotation, in consequence of the friction of the tides, as suggested by Hansen and Delaunay.[12]

Nevertheless, Laplace was right in attributing the acceleration, so far as it is a fact, to the variation of the eccentricity of the earth's orbit. It is proved that the greater the eccentricity of the terrestrial orbit, the greater is the disturbing action of the sun on the moon. Now, as the eccentricity has been decreasing for ages, the effect of the sun in disturbing the moon has been diminishing during that time. Consequently the attraction of the earth has had a more and more powerful effect on the moon, and has been continually diminishing the size of the lunar orbit. So that the moon's velocity has been gradually augmenting for many centuries to balance the increase of the earth's attraction. This secular increase in the moon's velocity is called the Acceleration, a name peculiarly appropriate at present, and which will continue to be so for a vast number of ages; because, as long as the earth's eccentricity diminishes, the moon's mean motion will be accelerated; but when the eccentricity has passed its minimum, and begins to increase, the mean motion will be retarded from age to age. The secular acceleration is now about $11''.9$, but its effect on the moon's place increases as the square of the time.[13] It is remarkable that the action of the planets, thus reflected by the sun to the moon, is much more sensible than their direct action either on the earth or moon. The secular diminution in the eccentricity, which has not altered the equation of the centre of the sun by eight minutes since the earliest recorded eclipses, has produced a variation of about $1^\circ\ 48'$ in the moon's longitude, and of $7^\circ\ 12'$ in her mean anomaly.[14]

The action of the sun occasions a rapid but variable motion in the nodes and perigee of the lunar orbit. Though the nodes recede during the greater part of the moon's revolution, and advance during the smaller, they perform their sidereal revolution in $6793^d\ 9^h\ 23^m\ 9^s.3$, or about 18.6 years; and the perigee accomplishes a revolution, called of the moon's apsides, in

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[11] See note 39, *Foreword, Bk. II.*

[12] *Ast. Soc. Monthly Notices*, vol. xxviii. p. 117. (Somerville's note.)

[13] In all investigations hitherto made with regard to the acceleration, it was tacitly assumed that the areas described by the radius vector of the moon were not permanently altered; that is to say, that the tangential disturbing force produced no disturbing effect. But Mr. Adams has discovered that, in consequence of the constant decrease in the eccentricity of the earth's orbit, there is a gradual change in the central disturbing force which affects the areal velocity, and consequently it alters the amount of the acceleration by a very small quantity, as well as the variation and the other periodic inequalities of the moon. On the latter, however, it has no permanent effect, because it affects them in opposite directions in very moderate intervals of time, whereas a very small error in the amount of the acceleration goes on increasing as long as the eccentricity of the earth's orbit diminishes, so that it would ultimately vitiate calculations of the moon's place for distant periods of time. This shows how complicated the moon's motions are, and what rigorous accuracy is required in their determination.

To give an idea of the labour requisite *merely* to *perfect* or *correct* the lunar tables, the moon's place was determined by observation at the Greenwich Observatory (see note 22, *Preliminary Dissertation*) in 6,000 different points of her orbit, each of which was compared with the same points calculated from Baron Plana's (see note 19, *Bk. II, Chap. X*) formulae, and to do that *sixteen computers* were constantly employed for *eight years*. Since the longitude is determined by the motions of the moon, the lunar tables are of the greatest importance. (Somerville's note.)

[14] *Mean anomaly*. The mean anomaly of a planet is its angular distance from the perihelion, supposing it to move in a circle. The true anomaly is the angular distance from the perihelion in its elliptical orbit. (Somerville note.)

$3232^d\ 13^h\ 48^m\ 29^s.6$, or a little more than nine years, notwithstanding its motion is sometimes retrograde and sometimes direct: but such is the difference between the disturbing energy of the sun and that of all the planets put together, that it requires no less than 109,830 years for the greater axis of the terrestrial orbit to do the same, moving at the rate of $11''.8$ annually. The form of the earth has no sensible effect either on the lunar nodes or apsides. It is evident that the same secular variation which changes the sun's distance from the earth, and occasions the acceleration in the moon's mean motion, must affect the nodes and perigee. It consequently appears, from theory as well as observation, that both these elements are subject to a secular inequality, arising from the variation in the eccentricity of the earth's orbit, which connects them with the acceleration, so that both are retarded when the mean motion is anticipated. The secular variations in these three elements are in the ratio of the numbers 3, 0.735, and 1; whence the three motions of the moon, with regard to the sun, to her perigee, and to her nodes, are continually accelerated, and their secular equations are as the numbers 1, 4.702, and 0.612. A comparison of ancient eclipses observed by the Arabs, Greeks, and Chaldeans, imperfect as they are, with modern observations, confirms these results of analysis. Future ages will develop these great inequalities, which at some most distant period will amount to many circumferences.[15] They are, indeed, periodic; but who shall tell their period? Millions of years must elapse before that great cycle is accomplished.

The moon is so near, that the excess of matter at the earth's equator occasions periodic variations in her longitude, and also that remarkable inequality in her latitude, already mentioned as a nutation in the lunar orbit, which diminishes its inclination to the ecliptic when the moon's ascending node coincides with the equinox of spring, and augments it when that node coincides with the equinox of autumn. As the cause must be proportional to the effect, a comparison of these inequalities, computed from theory, with the same given by observation, shows that the compression of the terrestrial spheroid, or the ratio of the difference between the polar and the equatorial diameters, to the diameter of the equator, is $\frac{1}{305}$. It is proved analytically, that, if a fluid mass of homogeneous matter, whose particles attract each other inversely as the squares of the distance, were to revolve about an axis as the earth does, it would assume the form of a spheroid whose compression is $\frac{1}{305}$. Since that is not the case, the earth cannot be homogeneous, but must decrease in density from its centre to its circumference. Thus the moon's eclipses show the earth to be round; and her inequalities not only determine the form, but even the internal structure of our planet; results of analysis which could not have been anticipated. Similar inequalities in the motions of Jupiter's satellites prove that his mass is not homogeneous. His equatorial diameter exceeds his polar diameter by about 6000 miles.

The phases of the moon, which vary from a slender silvery crescent soon after conjunction, to a complete circular disc of light in opposition, decrease by the same degrees till the moon is again enveloped in the morning beams of the sun. These changes regulate the returns of the eclipses. Those of the sun can only happen in conjunction, when the moon, coming between the earth and the sun, intercepts his light. Those of the moon are occasioned by the earth intervening between the sun and moon when in opposition. As the earth is opaque and nearly spherical, it throws a conical shadow on the side of the moon opposite to the sun, the axis of which passes through the centres of the sun and earth. The length of the shadow terminates at the

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[15] *Many circumferences.* There are 360 degrees, or 1,296,000 seconds in a circumference; and, as the acceleration of the moon only increases at the rate of eleven seconds in a century, it must be a prodigious number of ages before it accumulates to many circumferences. (Somerville's note.)

point where the apparent diameters[16] of the sun and earth would be the same. When the moon is in opposition, and at her mean distance, the diameter of the sun would be seen from her centre under an angle of 1918″.1. That of the earth would appear under an angle of 6908″.3. So that the length of the shadow is at least three times and a half greater than the distance of the moon from the earth, and the breadth of the shadow, where it is traversed by the moon, is about eight-thirds of the lunar diameter. Hence the moon would be eclipsed every time she is in opposition, were it not for the inclination of her orbit to the plane of the ecliptic, in consequence of which the moon, when in opposition, is either above or below the cone of the earth's shadow, except when in or near her nodes. Her position with regard to them occasions all the varieties in the lunar eclipses. Every point of the moon's surface successively loses the light of different parts of the sun's disc before being eclipsed. Her brightness therefore gradually diminishes before she plunges into the earth's shadow. The breadth of the space occupied by the penumbra[17] is equal to the apparent diameter of the sun, as seen from the centre of the moon. The mean duration of a revolution of the sun, with regard to the node of the lunar orbit, is to the duration of a synodic revolution[18] of the moon as 223 to 19. So that, after a period of 223 lunar months, the sun and moon would return to the same relative position with regard to the node of the moon's orbit, and therefore the eclipses would recur in the same order were not the periods altered by irregularities in the motions of the sun and moon. In lunar eclipses, our atmosphere, bends the sun's rays which pass through it all round into the cone of the earth's shadow. And as the horizontal refraction[19] or bending of the rays surpasses half the sum of the semidiameters of the sun and moon, divided by their mutual distance, the centre of the lunar disc, supposed to be in the axis of the shadow, would receive the rays from the same point of the sun, round all sides of the earth; so that it would be more illuminated than in full moon, if the greater portion of the light were not stopped or absorbed by the atmosphere. Instances are recorded where this feeble light has been entirely absorbed, so that the moon has altogether disappeared in her eclipses.

The sun is eclipsed when the moon intercepts his rays. The moon though incomparably smaller than the sun is so much nearer the earth, that her apparent diameter differs but little from his, but both are liable to such variations that they alternately surpass one another. Were the eye of a spectator in the same straight line with the centres of the sun and moon, he would see the sun eclipsed. If the apparent diameter of the moon surpassed that of the sun, the eclipse would be total. If it were less, the observer would see a ring of light round the disc of the moon, and the eclipse would be annular, as it was on the 17th of May, 1836, on the 15th of March, 1858, and on the 6th of March, 1867. If the centre of the moon should not be in a straight line joining the centres of the sun and the eye of the observer, the moon might only eclipse a part of the sun. The variation, therefore, in the distances of the sun and moon from the centre of the earth, and of the moon from her node at the instant of conjunction, occasions great varieties in the solar eclipses. Besides, the height of the moon above the horizon changes her apparent diameter, and may augment or diminish the apparent distances of the centres of the sun and moon, so that an eclipse of the sun may occur to the inhabitants of one country, and not to those of another. In this respect the solar eclipses differ from the lunar, which are the same for every part of the earth where the moon is above the horizon. In solar eclipses, the light reflected by the atmosphere diminishes the

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16 *Apparent diameter*. The diameter of a celestial body as seen from the earth. (Somerville's note.)
17 *Penumbra*. The shadow or imperfect darkness which precedes and follows an eclipse. (Somerville's note.)
18 *Synodic revolution of the moon*. The time between two consecutive new or full moons. (Somerville's note.)
19 *Horizontal refraction*. The light, in coming from a celestial object, is bent into a curve as soon as it enters our atmosphere; and that bending is greatest when the object is in the horizon. (Somerville's note.)

obscurity they produce. Even in total eclipses the higher part of the atmosphere is enlightened by a part of the sun's disc, and reflects its rays to the earth. The whole disc of the new moon is frequently visible from atmospheric reflection. In total solar eclipses the slender luminous arc that is visible for a few seconds before the sun vanishes and also before he reappears, resembles a string of pearls surrounding the dark edge of the moon; it is occasioned by the sun's rays passing between the tops of the lunar mountains; it occurs likewise in annular eclipses.

A phenomenon altogether unprecedented was seen during the total eclipse of the sun which happened on the 8th of July, 1842. The moon was like a black patch on the sky surrounded by the well known faint whitish light or corona about the eighth of the moon's diameter in breadth, which is supposed to be the solar atmosphere rendered visible by the intervention of the moon. In this whitish corona there appeared three rose-coloured flames like the teeth of a saw. Similar flames were also seen in the white corona of the total eclipse which took place in 1851, and a long rose-coloured chain of what appeared to be jagged mountains or sierras united at the base by a red band seemed to be raised into the corona by mirage; but there is no doubt that the corona and red phenomena belong to the sun.

Planets sometimes eclipse one another. On the l7th of May, 1737, Mercury was eclipsed by Venus near their inferior conjunction; Mars passed over Jupiter on the 9th of January, 1591; and on the 30th of October, 1825, the moon eclipsed Saturn. These phenomena, however, happen very seldom, because all the planets, or even a part of them, are very rarely seen in conjunction at once; that is, in the same part of the heavens at the same time. More than 2500 years before our era the five great planets were in conjunction. On the 15th of September, 1186, a similar assemblage took place between the constellations of Virgo and Libra; and in 1801 the Moon, Jupiter, Saturn, and Venus were united in the heart of the Lion. These conjunctions are so rare, that Lalande has computed that more than seventeen millions of millions of years separate the epochs of the contemporaneous conjunctions of the six great planets.

The motions of the moon have now become of more importance to the navigator and geographer than those of any other heavenly body, from the precision with which terrestrial longitude is determined by occultations of stars, and by lunar distances. In consequence of the retrograde motion of the nodes of the lunar orbit, at the rate of $3'10''.64$ daily, these points make a tour of the heavens in a little more than eighteen years and a half. This causes the moon to move round the earth in a kind of spiral, so that her disc at different times passes over every point in a zone of the heavens extending rather more than $5^\circ 9'$ on each side of the ecliptic. It is therefore evident that at one time or other she must eclipse every star and planet she meets within this space. Therefore the occultation of a star by the moon is a phenomenon of frequent occurrence. The moon seems to pass over the star, which almost instantaneously vanishes at one side of her disc, and after a short time as suddenly reappears on the other. A lunar distance is the observed distance of the moon from the sun, or from a particular star or planet, at any instant. The lunar theory is brought to such perfection, that the times of these phenomena, observed under any meridian, when compared with those computed for that of Greenwich, and given in the *Nautical Almanac*, furnish the longitude of the observer within a few miles.

From the lunar theory, the mean distance of the sun from the earth, and thence the whole dimensions of the solar system, are known; for the forces which retain the earth and moon in their orbits are respectively proportional to the radii vectores of the earth and moon, each being divided by the square of its periodic time. And, as the lunar theory gives the ratio of the forces, the ratio of the distances of the sun and moon from the earth is obtained. Hence it appears that the sun's mean distance from the earth is 384.26 times greater than that of the moon. The method of finding the absolute distances of the celestial bodies, in miles, is in fact the same with that employed in measuring the distances of terrestrial objects. From the extremities of a known base, the angles which the visual rays from the object form with it are measured; their sum subtracted from two right angles gives the angle opposite the base, therefore, by trigonometry, all the angles and sides of the triangle may be computed–consequently the distance of the object is found. The angle under which the base of the triangle is seen from the object is the parallax of that object. It evidently increases and decreases with the distance. Therefore the base must be very great indeed to be visible from the celestial bodies. The globe itself, whose dimensions are obtained by actual measurement, furnishes a standard of measures with which we compare the distances, masses, densities, and volumes of the sun and planets.