BOOK II

CHAPTER XIII

DATA FOR COMPUTING THE CELESTIAL MOTIONS

596. THE data requisite for computing the motions of the planets determined by observation for any instant arbitrarily assumed as the epoch or origin of the time, are

The masses of the planets;
Their mean sidereal motions for a Julian year of 365.25 days;
The mean distances of the planets from the sun;
The ratios of the eccentricities to the mean distances;
The inclinations of the orbits on the plane of the ecliptic;
The longitudes of the perihelia;
The longitudes of the ascending nodes on the ecliptic;
The longitudes of the planets.

Masses of the Planets

597. Satellites afford the means of ascertaining the masses of their primaries; the masses of such planets as have no satellites are found from a comparison of their inequalities determined by analysis, with values of the same obtained from numerous observations. The secular inequalities will give the most accurate values of the masses, but till they are perfectly known the periodic variations must be employed. On this account there is still some uncertainty as to the masses of several bodies. It is only necessary to know the ratio of the mass of each planet to that of the sun taken as the unit; the masses are consequently expressed by very small fractions.

598. If the time of a sidereal revolution of a planet m, whose mean distance from the sun is a, p the ratio of the circumference to the diameter, and $m = \sqrt{m+S}$ the sum of the sun and planet, by article 383,

$$T = \frac{2p \, . \, a^{\frac{3}{2}}}{\sqrt{m}}.$$

From this expression the masses of such planets as have satellites may be obtained.

Suppose this equation relative to the earth, and that the mass of the earth is omitted when compared with that of the sun, it then becomes

$$T = \frac{2p\ .\ a^{\frac{3}{2}}}{\sqrt{S}}.$$

Again, let $m = m + m'$ the sum of the masses of a planet and of its satellite m', T' being the time of a sidereal revolution of the planet at the mean distance a' from the sun, then

$$T' = \frac{2p\ .\ a'^{\frac{3}{2}}}{\sqrt{m + m'}};$$

and dividing the one by the other the result is,

$$\frac{m + m'}{S} = \frac{a'^3}{a^3} \cdot \frac{T^2}{T'^2}.$$

If the values of T, T', a, and a', determined from observation, be substituted in this expression, the ratio of the sum of the masses of the planet and of its satellite to the mass of the sun will be obtained; and if the mass of the satellite be neglected when compared with that of its primary, or if the ratio of these masses be known, the preceding equation will give the ratio of the mass of the planet to that of the sun. For example,

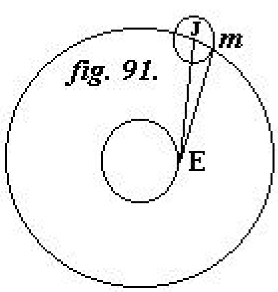

fig. 91.

599. Let m be the mass of Jupiter [fig. 91], that of his satellite being omitted, and let the mass of the sun be taken as the unit, then

$$m = \frac{a'^3}{a^3} \cdot \frac{T^2}{T'^2}.$$

J*m* the mean radius of the orbit of the fourth satellite at the mean distance of the earth from the sun taken as the unit, is seen under the angle JE*m*=2580″.579. The radius of the circle reduced to seconds is 206264″.8; hence the mean radii of the orbit of the fourth satellite and of the terrestrial orbit are in the ratio of these two numbers. The time of a sidereal revolution of the fourth satellite is 16.6890 days, and the sidereal year is 365.2564 days, hence

$$\begin{aligned} a &= 206264.8 \\ a' &= 2580.58 \\ T &= 365.2564 \\ T' &= 16.6890. \end{aligned}$$

With these data it is easy to find that the mass of Jupiter is

$$m = \frac{1}{1066.09}.$$

The sixth satellite of Saturn accomplishes a sidereal revolution in 15.9453 days; the mean radius of its orbit, at the mean distance of the planet, is seen from the sun under an angle of 179″; whence the mass of Saturn is

$$\frac{1}{3359.40}.$$

By the observations of Sir William Herschel[1] the sidereal revolutions of the fourth satellite of Uranus are performed in 13.4559 days, and the mean radius of its orbit seen from the sun at the mean distance of the planet is 44”.23. With these data the mass of Uranus is found to be

$$\frac{1}{19{,}504}.$$

600. This method is not sufficiently accurate for finding the mass of the Earth, on account of the numerous inequalities of the Moon. It has already been observed, that the attraction of the Earth on bodies at its surface in the parallel where the square of the sine of the latitude is $\frac{1}{3}$, is nearly the same as if its mass were united at its centre of gravity. If R be the radius of the terrestrial spheroid drawn to that parallel, and m its mass, this attraction will be

$$g = \frac{m}{R^2}; \text{ whence } m = g \,.\, R^2.$$

Then, if a be the mean distance of the Sun from the Earth, T the duration of the sidereal year,

$$T = \frac{2\pi \,.\, a^{\frac{3}{2}}}{\sqrt{S}};$$

and, by division,

$$\frac{m}{S} = \frac{g \,.\, R^2 T^2}{4\pi \,.\, a^3}.$$

R, g, T, and a, are known by observation, therefore the ratio of the mass of the Earth to that of the Sun[2] may be found from this expression.

The sine of the solar parallax at the mean distance of the sun from the earth,[3] and in the latitude in question, is

$$\sin P = \frac{R}{a} = \sin 8''.75;$$

the attraction of the Earth, and the terrestrial radius in the same parallel, are

$$g = 2\times 16.1069 = 32.2138$$
$$R = 2089870,$$

and the sidereal year is

$$T = 31558152''.9$$

with these data the mass of the earth is computed to be[4]

$$\frac{1}{337{,}103},$$

the mass of the sun being unity. This value varies as the cube of the solar parallax compared with that adopted.

601. The compression of the three larger planets, and the ring of Saturn, probably affect the values of the masses computed from the elongations of their satellites; but the comparison of numerous well chosen observations, with the disturbances determined from theory, will ultimately give the masses of all the planets with great accuracy.

The action of each disturbing body adds a term of the form $m'dv'$ to the longitude, so that the longitude of m at any given instant in its troubled orbit, is

$$v + m'dv' + m''dv'' + \&c.$$

[where][5] $v,\ dv',\ dv''$, are susceptible of computation from theory; and as they are given by the Tables of the Motions of the Planets, the true longitude of m is

$$v + m'dv' + m''dv'' + \&c. = L\,.$$

When this formula is compared[6] with a great number of observations, a series of equations,

$$m'dv' + m''dv'' + \&c. = L - v,$$
$$m'dv_2' + m''dv_2'' + \&c. = L' - v_2,$$
$$+ \&c. = \&c.$$

are obtained, where m', m'', &c., are unknown quantities, and by the resolution of these the masses of the planets may be estimated by the perturbations they produce,

602. As there are ten planets,[7] ten equations would be sufficient to give their masses, were the observed longitudes and the computed quantities v, dv', dv'', &c., mathematically exact; but as that is far from being the case, many hundreds of observations made on all the planets must be employed to compensate the errors. The method of combining a series of equations more numerous than the unknown quantities they contain, so as to determine these quantities with all possible accuracy, depends on the theory of probabilities, which will be explained afterwards. The powerful energy exercised by Jupiter on the four new planets in his immediate vicinity occasions very great inequalities in the motions of these small bodies, whence that highly distinguished mathematician, M. Gauss,[8] has obtained a value for the mass of Jupiter, differing considerably from that deduced from the elongation of his satellites, it cannot however be regarded as conclusive till the perturbations of these small planets are perfectly known.

603. The mass of Venus is obtained from the secular diminution in the obliquity of the Ecliptic. The plane of the terrestrial equator is inclined to the plane of the ecliptic at an angle of $23^{\circ}\ 28'\ 47''$ nearly, but this angle varies in consequence of the action of the planets. A series of tolerably correct observations of the Sun's altitude at the solstices chiefly by the Chinese and Arabs, have been handed down to us from the year 1100 before Christ, to the year 1473 of the Christian era; by a comparison of these, it appears that the obliquity was then diminishing, and it is still decreasing it the rate of $50''.2$ in a century. From numerous observations on the obliquity of the ecliptic made by Bradley[9] about a hundred years ago, and from later observations by Dr. Maskelyne,[10] Delambre[11] determined the maximum of the inequalities produced by the action of Venus, Mars, and the Moon, on the Earth, and by comparing these observations with the analytical formulae, he obtained nearly the same value of the mass of Venus, whether he deduced it from the joint observations of Bradley and Maskelyne, or from the observations of each separately. From this correspondence in the values of the mass of Venus, obtained from these different sets of observations, there can be little doubt that the secular diminution in the obliquity of the ecliptic is very nearly $50''.2$, and the probability of accuracy is greater as it agrees with the observations made by the Chinese and Arabs so many centuries ago. Notwithstanding doubts still exist as to the mass of Venus.

604. The mass of Mars has been determined by the same method, though with less precision than that of Venus, because its action occasions less disturbance in the Earth's motions, for it is evident that the masses of those bodies that cause the greatest disturbance will be best known. The action of the new planets is insensible, and that of Mercury has a very small influence on the motions of the rest. An ingenious method of finding the mass of that planet has been adopted by Laplace, although liable to error.

605. Because mass is proportional to the product of the density and the volume, if m, m', be the masses of any two planets of which r, r', are the densities, and V, V', the volumes, then

$$m : m' :: r.V : r'.V'.$$

But as the planets differ very little from spheres, their volumes may be assumed proportional to the cubes of their diameters; hence if D, D', be the diameters of m, and m',

$$m : m' :: r.D^3 : r'.D'^3;$$

whence

$$\frac{r}{r'} = \frac{D'^3}{D^3} \cdot \frac{m}{m'}. \qquad \textbf{(201)}$$

The apparent diameters of the planets have been measured so that D and D' are known; this equation will therefore give the densities if the masses be known, and *vice versâ*.

By comparing the masses of the Earth, Jupiter, and Saturn, with their volumes, Laplace found that the densities of these three planets are nearly in the inverse ratio of their mean distances from the sun, and adopting the same hypothesis with regard to Mercury, Mars, and Jupiter, he obtained the preceding values of the masses of Mars and Mercury, which are found nearly to agree with those determined from other data. Irradiation, or the spreading of the light round the disc of a planet, and other difficulties in measuring the apparent diameters, together with the uncertainty of the hypothesis of the law of the densities, makes the values of the masses obtained in this way the more uncertain, as the hypothesis does not give a true result for the masses of Venus and Saturn. Fortunately the influence of Mercury on the solar System is very small.

606. The mass of the Sun being unity, the masses of the planets are,[12]

Mercury.	$\frac{1}{2,025,810}$
Venus	$\frac{1}{405,871}$
The Earth	$\frac{1}{354,936}$
Mars	$\frac{1}{2,546,320}$
Jupiter.	$\frac{1}{1,070.5}$
Saturn	$\frac{1}{3,512}$
Uranus.	$\frac{1}{17,918}$

Densities of the Planets

607. The densities of bodies are proportional to the masses divided by the volumes, and when the masses are spherical, their volumes are as the cubes of their radii; as the sun and planets are nearly spherical, their densities are therefore as their masses divided by the cubes of their radii; but the radii must be taken in those parallels of latitude, the squares of whose sines are $\frac{1}{3}$.

The mean apparent semidiameters of the Sun and Earth at their mean distance are,

Sun	961″
The Earth	8″.6

The radius of Jupiter's spheroid in the latitude in question, when viewed at the mean distance of the earth from the sun, is 94″.344; and the corresponding radius of Saturn at his mean distance from the sun is 8″.1. Whence the densities are,

Sun	1
The Earth	3.9326
Jupiter	.99239
Saturn	.59496

Thus the densities decrease with the distance from the sun; however that of Uranus does not follow this law, being greater than that of Saturn, but the uncertainty of the value of its apparent diameter may possibly account for this deviation.

Intensity of Gravitation at the Surfaces of the Sun and Planets

608. Let g and g' represent the force of gravity at the surfaces of two bodies m and m', whose apparent diameters are D and D'. If the bodies be spherical and without rotation, the force of gravity at their equators will be as their masses divided by the squares of their diameters; hence

$$g = g' \cdot \frac{m}{m'} \cdot \frac{D'^2}{D^2}.$$

Because the masses, apparent diameters, and the intensity of gravity at the terrestrial equator are known, g, the intensity of the gravitating force at the equator of any other body may be found; and as the rotation of the sun and planets is determined by observation, their centrifugal forces, and consequently the intensity of gravitation at their surfaces may be computed. With the preceding values of the masses and apparent diameters it will be found, that if the weight of a body at the terrestrial equator be the unit, the same body transported to the equator of Jupiter, would weigh 2.716; but this would be diminished by about a ninth, on account of the centrifugal force. The same body would weigh 27.9 at the sun's equator, and a body at the sun's equator would fall through 448.39 feet in the first second of its descent, that would only fall through 16.0436 feet at the earth's equator.

To determine the fall of bodies at the surfaces of the sun and planets was hopeless till Newton's immortal discovery connected us with remote worlds.

609. The mean sidereal motions of the planets in a Julian year of 365.25 days are the second data.

When the sun is in the tropics his declination is a maximum, and equal to the obliquity of the ecliptic; the time at which that happens is found by observing his declination at noon for several days before and after the instant of a solstice, so that an equation can be formed between the time and the declination, which is sufficiently exact for a few days. If the differential of the declination in this equation be made zero, the instant of the solstice and the obliquity of the ecliptic will be obtained. The instant of the equinoxes is determined in the same manner, only that in the equation between the time and the declination, the declination is made zero, for in these points the sun is in the plane of the equator.[13] The length of the year is determined by comparing together the time of the sun's being in either equinox, or in either tropic, with the time of his being in the same point for another year distant from the former by a long period; the interval reckoned in days and parts of a day, divided by the number of years elapsed, will give the true length of the year; and the greater the interval, the more correct will it be. The length of the year however, like all astronomical data, was determined by successive approximations, but it was very early known to be 365.25 days.

The Julian year being known, if the synodic revolutions of the planets be known, their mean motion for any given interval may be found.

610. The longitude of an inferior planet in superior[14] conjunction, or of a superior planet in opposition, is the same as if viewed from the centre of the sun. The synodical revolution of the planet, which is the interval between two conjunctions, or two oppositions, may be ascertained by observation, and from thence its periodic time. Let T be the synodic revolution of a planet, P its periodic time, then

$$P : 365.25 :: 360^0 : 360^0 \pm a,$$

the angle described by the planet in 365.25 days. If it be an inferior planet, its angular motion will be greater than that of the earth; hence the angle described in 365.25 days is equal to 360^0 plus the angle gained by the planet on the earth, or $360^0 + a$. But if it be a superior planet, its angular velocity being less than that of the earth, the angle described in a Julian year is $360^0 - a$. But these angles are as the times in which they are described, therefore

$$360^0 : 360^0 \pm a :: T : 365.25 \pm T;$$

hence

$$P : 365.25 :: T : 365.25 \pm T,$$

and

$$P = \frac{365.25 \times T}{365.25 \pm T}.$$

As the synodic revolutions are known, the sidereal revolutions of the planets are as follow.

	Days
Mercury	87.9705
Venus	224.7
The Earth	365.2564

Mars	686.99
Vesta	1,592.69
Juno	1,331.
Ceres	1,681.42
Pallas	1,686.56
Jupiter	4,332.65
Saturn	10,759.4
Uranus	30,687.5

Whence it will be found by simple proportion that the mean sidereal motions of the planets in a Julian year of 365.2564 days, or the values of n, n', &c., are

Mercury	53,831,034″.99
Venus	2,106,644″.82
The Earth	12,995,977″.74
Mars	689,051″.63
Vesta	355,681″.17
Juno	297,216″.21
Ceres	281,531″.00
Pallas	280,672″.32
Jupiter	109,256″.78
Saturn	43,996″.13
Uranus	15,425″.64

These have been determined by approximation, continually corrected by a long series of observations on the oppositions and conjunctions of the planets.

Mean Distances of the Planets, or Values of a, a', a'', &c.

611. The mean distances are obtained from the mean motions of the planets: for, assuming the mean distance of the earth from the sun as the unit, Kepler's law of the squares of the periodic times being as the cubes of the mean distances, gives the following values of the mean distances of the planets from the sun.[15]

	[A.U.][16]
Mercury	0.3870981
Venus	0.7233316
The Earth	1.0000000
Mars	1.5236923
Vesta	2.3678700
Juno	2.6690090
Ceres	2.7672450
Pallas	2.7728860
Jupiter	5.2011524

Saturn	9.5379564
Uranus	19.1823927

Ratio of the Eccentricities to the Mean Distances, or Values of e, e', *&c. for 1801*

612. The eccentricity of an orbit is found by ascertaining that heliocentric longitude of the planet at which it is moving with its mean angular velocity, for there the increments of the true and mean anomaly are equal to one another, and the equation of the centre, or difference between the mean and true anomaly is a maximum, and equal to half the eccentricity. By repeating this process for a series of years, the effects of the secular variations will become sensible, and may be determined; and when they are known, the eccentricity may be determined for any given period. The values of e, e', e'', &c., for 1801, are

Mercury	0.20551494
Venus	0.00686074
The Earth	0.01685318
Mars	0.09330700
Vesta	0.08913000
Juno	0.25784800
Ceres	0.07843900
Pallas	0.24164800
Jupiter	0.04816210
Saturn	0.05615050
Uranus	0.04661080

Inclinations of the Orbits on the Plane of the Ecliptic, in 1801

613. When the earth is in the line of a planet's nodes, if the planet's elongation from the sun and its geocentric latitude be observed, the inclination of the orbit may be found; for the sine of the elongation is to the radius, as the tangent of the geocentric latitude to the tangent of the inclination. If the planet be 90° distant from the sun, the latitude observed is just equal to the inclination. By this method Kepler determined the inclination of the orbit of Mars. The secular inequalities become sensible after a course of years. The values of f, f', f'', &c. were in 1801

		°	′	″
	Mercury	7	0	9.1
	Venus	3	23	28.5
	Mars	1	51	6.2
1820	Vesta	7	8	9.0
1820	Juno	13	4	9.7
1820	Ceres	10	37	26.2
1820	Pallas	34	34	55.0
	Jupiter	1	18	51.3
	Saturn	2	29	35.7
	Uranus	0	46	28.4

Longitudes of the Perihelia

614. The angular velocity of a body is least in aphelion, and greatest in perihelion; consequently, if its longitude be observed when the increments of the angular velocity are greatest or least, these points will be in the extremities of the major axis: if these be really the two observed longitudes, the interval between them will be exactly half the time of a revolution, a property belonging to no other diameter in the ellipse. As it is very improbable that the observations should differ by 180^{o}, they require a small correction to reduce them to the true times and longitudes. On this principle the longitudes of the perihelia may be determined, and if the observations be continued for a series of years, their secular motions will be obtained, whence their places may be computed for any epoch. The longitude of the perihelion is the distance of the perihelion from the ascending node estimated on the orbit, plus the longitude of the node. In the beginning of 1801, the values of v, v', v'', &c., were,

		°	′	″
	Mercury	74	21	46.8
	Venus	128	43	53.0
	The Earth	99	30	4.8
	Mars	332	23	56.4
1820	Vesta	249	33	24.2
1820	Juno	53	33	46.0
1820	Ceres	147	7	31.1
1820	Pallas	121	7	4.3
	Jupiter	11	8	34.4
	Saturn	89	9	29.5
	Uranus	167	30	23.7

Longitudes of the Ascending Nodes

615. When a planet is in its nodes, it is in the plane of the ecliptic; its longitude is then the same with the longitude of its node, and its latitude is zero. The place of the nodes may therefore be found by a series of observations, and if they be continued long enough, their secular motions will be obtained; whence their positions at any time may be computed. In the beginning of 1801 the values of q, q', q'', &c., were,

		°	′	″
	Mercury	45	57	30.9
	Venus	74	54	12.9
	Mars	48	0	3.5
1820	Vesta	103	13	18.2
1820	Juno	171	7	40.4
1820	Ceres	80	41	24.0
1820	Pallas	172	39	26.8

Jupiter	98	26	18.9
Saturn	111	56	37.3
Uranus	72	59	35.4

616. Mean longitudes of the planets on the 1st January, 1801, at midnight, or values of $\in$, $\in'$, $\in''$, &c.

		°	′	″
	Mercury	163	56	26.9
	Venus	10	44	21.6
	The Earth	100	9	12.9
	Mars	64	6	59.9
1820 1st Jan at noon	Vesta	278	30	0.4
1820 1st Jan at noon	Juno	200	16	19.1
1820 1st Jan at noon	Ceres	123	16	11.9
1820 1st Jan at noon	Pallas	108	24	57.9
	Jupiter	112	12	51.3
	Saturn	135	19	5.5
	Uranus	177	48	1.1

All the longitudes are estimated from the mean equinox of spring, the epoch being the 1st January, 1801.

617. With these data the motions of the planets are computed; they are, however, only approximate, since each element is determined independently of the rest; whereas they are so connected, that their values ought to be determined simultaneously by equations of condition formed from thousands of observations.

618. Elements of the orbits of the three comets belonging to the solar system.

Halley's Comet of 1682[17]

Period of revolution 76 years, nearly. Instant of passage at perihelion 1835, October 31st 2.

Half the greater axis	17.98355
Eccentricity	0.967453
Longitude of perihelion on orbit	304° 34′ 19″
Longitude of ascending node	55 6 59
Inclination	17 46 50

Motion Retrograde

Encke's Comet of 1819[18 19]

Period of Revolution 1205.55 days.[20] Passage at perihelion 1829, January 10th, 573.

Mean diurnal motion	1069″.557

Half the greater axis	2.224346
Eccentricity	0.8446862
Longitude of perihelion	157° 18′ 35″
Longitude of ascending node	334 24 15
Inclination	13 22 34

Claussen and Gambart's Comet of 1825

Period of revolution 6.7 years.[21] Passage at perihelion 1832, November 27th, 4808.

Half the greater axis	3.53683
Eccentricity	0.7517481
Longitude of perihelion	109° 56′ 45″
Longitude of ascending node	248 12 24
Inclination	13 13 13

The computation, in the next Chapter, of the perturbations of Jupiter and Saturn will be sufficient to show the method of finding their numerical values, especially as there are many peculiar to these two planets.

Notes

1 See note 52, *Preliminary Dissertation.*
2 The capitalization of planetary names is not consistent in the text. We retain the 1st edition assignments.
3 See preceding note.
4 The modern value is closer to $\frac{1}{333{,}000}$ or 3.33×10^{-5}.
5 This *reads* $v_{/}$, in the 1st edition.
6 This *reads* "composed" in the 1st edition (published erratum).
7 Including Earth, at the time of writing there are actually eleven "planets." These are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus and the recently discovered "telescopic planets," the asteroids Ceres, Pallas, Juno, and Vesta.
8 Gauss, (Johann) Carl Friedrich, 1777-1855, mathematician, born in Brunswick, Germany. He was professor of mathematics and director of the observatory at Göttingen and considered one of the greatest mathematicians of all time. Gauss wrote the first modern book on number theory and pioneered a number of mathematical applications in gravitation, magnetism, and electricity. His astronomical work included careful calculations of the orbits of the asteroids Ceres and Pallas using his method of least squares (see also note 60, *Bk. II, Chap. XIV*). Gauss was also interested in the shape of the earth and invented a heliotrope to increase the accuracy of surveying measurements. His *Theoria motus corporum coelestium in sectionibus conicis Solem ambientium* (1809) is a two volume treatise on celestial motion. It treats conic sections, elliptical orbits, and methods for refining planetary orbits. The unit of magnetic induction is named after him.
9 See note 38, *Preliminary Dissertation.*
10 See note 55, *Bk. II, Chap. VI.*
11 See note 54, *Preliminary Dissertation.*
12 The modern values of the reciprocals of these ratios to 4 significant figures (with Somerville's values also rounded to 4 significant figures in parentheses for comparison) are: Mercury 602,700 (2,026,000), Venus 408,500 (405,900), Earth 333,000 (355,000), Mars 3,097,000 (255,000), Jupiter 1,047 (1,071), Saturn 3,502 (3,512), Uranus

22,910 (17,920). Note that Somerville does not use comma separators in the presentation of her data in the 1st edition.

[13] This *reads* "ecliptic" in the 1st edition (published erratum).

[14] This modifier is omitted in the 1st edition (published erratum).

[15] The modern values of these distances in AU (with Somerville's values in parentheses for comparison) are: Mercury 0.38 (0.39), Venus 0.72 (0.72), Earth 1.00 (1.00), Mars 1.52 (1.52), Vesta 2.36 (2.37), Juno 2.67 (2.67), Ceres 2.77 (2.77), Pallas 2.77 (2.77), Jupiter 5.20 (5.20), Saturn 9.54 (9.54), Uranus 19.218 (19.182).

[16] The 1st edition text does not use the AU designation. An AU (Astronomical Unit) is the mean distance of the Earth

[17] See note 55, *Preliminary Dissertation*.

[18] *Encke*. Somerville spells the name "Enke" in the 1st edition.

[19] Encke, Johann Franz, 1791-1865, astronomer, born in Hamburg, Germany. Encke made the first accurate calculation of solar distance and calculated the solar parallax with a precision close to the modern value. He directed the Seeberg Observatory (1822-5) and later became director of the Observatory at Berlin University. In 1819 Encke measured the period of the comet that bears his name. He also discovered the outer ring of Saturn, known now as Encke's Division. Encke also developed new methods for the determination of the orbits of the newly discovered small planets or asteroids.

[20] This *reads* 1203.687 days in the 1st edition (published erratum).

[21] This *reads* 6,7 years in the 1st edition.