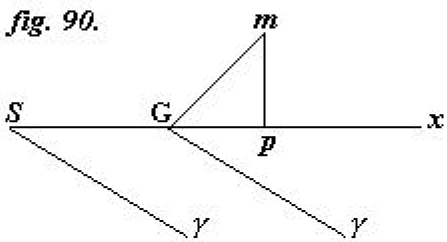
BOOK II

CHAPTER XII

PERTURBATIONS IN THE MOTIONS OF THE PLANETS OCCASIONED BY THE ACTION OF THEIR SATELLITES

594. THE common centre of gravity of a planet and its satellites very nearly describes an ellipse round the sun. If that orbit be considered to be the orbit of the planet itself, the respective positions of the satellites with regard to each other, and to the sun, will give that of the planet with regard to their common centre of gravity, and consequently the perturbations produced by the satellites on their primary.

fig. 90.



Let G, fig. 90, be the common centre of gravity of a planet, and of its satellites,[1] S the sun, g the equinoctial point, and $\overline{x}$, $\overline{y}$, $\overline{z}$, the co-ordinates of G, so that $S\mathrm{G}=\overline{x}$, and $\overline{z}$ perpendicular to the plane of the orbit. Then if x, y, z, be the co-ordinates of a satellite m, and $v=gS\mathrm{G}$, $U=g\mathrm{G}m$, the longitudes of G and m; it is evident that $\mathrm{G}p=x-\overline{x}$, and r being the radius Gm,

$$\mathrm{G}p=x-\overline{x}=r\,.\cos(U-v);$$

hence, if $\sum m$ be the sum of the masses of the satellites, and P that of their primary,

$$\sum m\,.\,x=\overline{x}\,.\,P+\sum m\,.\,r\cos(U-v),$$

or

$$\sum mx=\overline{x}\,.\,P+mr\,.\cos(U-v)+m'r'\,.\cos(U-v')+\&\text{c.}$$

In the same manner

$$\sum my=\overline{y}\,.\,P+mr\,.\sin(U-v)+m'r'\,.\sin(U-v')+\&\text{c.}$$

[and]

$$\sum mz=\overline{z}P+m\,.\,rs+m'\,.\,r's'+\&\text{c.}$$

s, s', s'', &c., being the latitudes of the satellites above the orbit of their common centre of gravity. But by the property of the centre of gravity,

$$\sum m\,.\,x=0,\ \sum m\,.\,y=0,\ \sum m\,.\,z=0;$$

consequently,

$$0 = \overline{x}\,.\,P + mr\,.\cos\left(U - v\right) + \&\text{c.}$$
$$0 = \overline{y}\,.\,P + mr\,.\sin\left(U - v\right) + \&\text{c.}$$
$$0 = \overline{z}\,.\ P + mr\,.\,s + m'r's' + \&\text{c.}$$

By article 353 the centre of gravity is urged in a direction parallel to the co-ordinates, by the forces

$$-\left(P + \Sigma m\right)\overline{x};\quad \frac{-\left(P + \Sigma m\right)\overline{y}}{\overline{r}};\quad \frac{-\left(P + \Sigma m\right)\overline{z}}{\overline{r}}.$$

[where] $\overline{r} = S\text{G},$ the radius vector of the centre of gravity. These forces vary very nearly as $\overline{x}$, $\frac{\overline{y}}{\overline{r}}$, and $\frac{\overline{z}}{\overline{r}}$; therefore the perturbations in the radius vector *S*G are very nearly proportional to $\overline{x}$, that is, to

$$-\frac{m}{P}\cdot r\cos\left(U - v\right) - \frac{m'}{P}\cdot r'\cos\left(U - v'\right) - \&\text{c.}$$

The perturbations in longitude are nearly proportional to

$$-\frac{m}{P}\cdot r\sin\left(U - v\right) - \frac{m'}{P}\cdot\frac{r'}{\overline{r}}\sin\left(U - v'\right) - \&\text{c.};$$

and those in latitude to[2]

$$-\frac{m}{P}\cdot\frac{rs}{\overline{r}} - \frac{m'r's'}{\overline{r}P} - \&\text{c.}$$

The masses of Jupiter's satellites compared with the mass of that planet are so small, and their elongations seen from the sun subtend so small an angle, that the perturbations produced by them in Jupiter's motions are insensible; and there is reason to believe this to be the case also with regard to Saturn and Uranus.

595.[3] But the Earth is sensibly troubled in its motions by the Moon, her action produces the inequalities[4]

$$d\,r = -\frac{m}{E}\cdot r\cos\left(U - v\right)$$
$$d\,v = -\frac{m}{E}\cdot\frac{r}{\overline{r}}\sin\left(U - v\right)$$
$$d\,s = -\frac{m}{E}\cdot\frac{r}{\overline{r}}\cdot s;$$

or, more correctly,

$$d r = -\frac{m}{E+m} \cdot r \cos\left(U - v\right)$$
$$d v = -\frac{m}{E+m} \cdot \frac{r}{\overline{r}} \sin\left(U - v\right) \qquad \textbf{(200)}$$
$$d s = -\frac{m}{E+m} \cdot \frac{r}{\overline{r}} \cdot s;$$

in the radius vector, longitude and latitude of the Earth, E and m being the masses of the Earth and Moon.

Notes

[1] In keeping with earlier use we italicize S in this edition. The 1st edition text *reads* S.
[2] The denominator in the third term *reads* $\overline{r}$ in the 1st edition (published erratum).
[3] This article is numbered 495 in the 1st edition.
[4] The third equation *reads* $d s = -\frac{m}{E} \cdot \frac{r}{r} \cdot s$ in the 1st edition.