BOOK II - UNIVERSAL GRAVITATION

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FOREWORD[1]

Gravitation

IT has been proved by Newton, that a particle of matter placed without the surface of a hollow sphere is attracted by it in the same manner as if the mass of the hollow sphere, or the whole matter it contains, were collected into one dense particle in its centre. The same is therefore true of a solid sphere, which may be supposed to consist of an infinite number of concentric hollow spheres, or shells, arranged around the same centre, like the coats of an onion. This, however, is not the case with a spheroid;[2] but the celestial bodies are so nearly spherical, and at such remote distances from one another, that they attract and are attracted as if each were condensed into a single particle situate in its centre of gravity[3]—a circumstance which greatly facilitates the investigation of their motions.

Newton has shown that the force which retains the moon in her orbit is the same with that which causes heavy substances to fall at the surface of the earth. If the earth were a sphere, and at rest, a body would be equally attracted, that is, it would have the same weight at every point of its surface, because the surface of a sphere is everywhere equally distant from its centre. But, as our planet is flattened at the poles, and bulges at the equator, the weight of the same body gradually decreases from the poles, where it is greatest, to the equator, where it is least. There is, however, a certain mean[4] latitude, or part of the earth intermediate between the pole and the equator, where the attraction of the earth on bodies at its surface is the same as if it were a sphere;[5] and experience shows that bodies there fall through 16.0697 feet in a second. The mean distance[6] of the moon from the earth is about sixty times the mean radius[7] of the earth. When the number 16.0697 is diminished in the ratio (or relation) of 1 to 3600, which is the square of the moon's distance[8] from the earth's centre, estimated in terrestrial radii, it is found to be exactly

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[1] The material in this and the forewords to Books I, III and IV is extracted from the 10th and last edition of Mary Somerville's *On the Connexion of the Physical Sciences*, (corrected and revised by Arabella B. Buckley), p. 4-106, London : John Murray, 1877.

[2] *spheroid.* A solid body which sometimes has the shape of an orange; it is then called an oblate spheroid, because it is flattened at the poles. Such is the form of the earth and the planets. When, on the contrary, it is drawn out at the poles like an egg, it is called a prolate spheroid. (Somerville's note.)

[3] *centre of gravity.* A point in every body, which if supported, the body will remain at rest in whatever position it may be placed. About that point all the parts exactly balance one another. (Somerville's note.)

[4] *Mean* quantities are such as are intermediate between others that are greater or less. (Somerville's note.)

[5] The attraction of a sphere on an external body is the same as if its mass were collected into one heavy particle at its centre of gravity, and the intensity of its attraction diminishes as the square of its distance from the external body increases. (Somerville's note.)

[6] The *mean distance* of a planet from the centre of the sun, or of a satellite from the centre of its planet, is equal to half the sum of its greatest and least distances, and consequently, is equal to half the major axis of its orbit. (Somerville's note.)

[7] The *mean radius of the earth* is the mean distance from the centre to the surface of the earth. It is intermediate between the distances measured from the pole to the centre and from the equator to the centre. (Somerville's note).

[8] In order to avoid large numbers, the mean radius of the earth is taken for unity: then the mean distance of the moon is expressed by 60; and the square of that number is 3600, or 60 times 60. (Somerville's note.)

the space the moon would fall through in the first second of her descent to the earth were she not prevented by the centrifugal force[9] arising from the velocity with which she moves in her orbit. The moon is thus retained in her orbit by a force having the same origin, and regulated by the same law, with that which causes a stone to fall at the earth's surface. The earth may, therefore, be regarded as the centre of a force which extends to the moon; and, as experience shows that the action and reaction[10] of matter are equal and contrary, the moon must attract the earth with an equal and contrary force. Nevertheless, since the earth's mass is so much greater than the mass of the moon, this equal force will only draw it over a comparatively small space.

Newton also ascertained that a body set in motion or projected in space, if attracted by a force proceeding from a fixed point, will move in a conic section[11] with an intensity inversely as the square of the distance;[12] but that any deviation from that law will cause it to move in a curve of a different nature. Kepler[13] found, by direct observation, that the planets describe ellipses, or oval paths, round the sun.[14] Later observations show that some comets also move in ellipses, but the greater part seem to move in parabolas, while others move in hyperbolas. All these are conic sections. It consequently follows that the sun attracts all the planets and comets inversely as the square of their distances from its centre; the sun, therefore, is the centre of a force extending indefinitely in space, and including all the bodies of the system in its action.

Kepler also deduced from observation that the squares of the periodic times of the planets, or the times of their revolutions round the sun, are proportional to the cubes of their mean distances from its centre. Hence the intensity of gravitation of all the bodies towards the sun is the same at equal distances; and if the planets and comets were at equal distances from the sun, and left to the effects of gravity, they would arrive at his surface at the same time.[15] The

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[9] *Centrifugal force.* A term formerly, but erroneously used to express the tendency of a revolving body to fly from the centre of attraction round which it revolves. The real explanation of this fact is found in Newton's law that *a body continues in its state of rest or uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state.* The true tendency of any body moving in space is therefore to proceed in a straight line, and it will continue to do so unless the direction of its motion is altered by the attraction of any other body. (Somerville's note.)

[10] *Action and reaction.* When motion is communicated by collision or pressure, the action of the body which strikes is returned with equal force by the body which receives the blow. (Somerville's note.)

[11] *Conic sections.* Lines formed by any plane cutting a cone. When the axis is perpendicular to the base, the solid is a right cone. If a right cone with a circular base be cut at right angles to the base by a plane passing through the apex, the section will be a triangle. If the cone be cut through both sides by a plane parallel to the base, the section will be a circle. If the cone be cut slanting quite through both sides the section will be an ellipse. If the plane be cut parallel to one of the sloping sides the section will be a parabola. And if the plane cut only one side of the cone, and be not parallel to the other, the section will be a hyperbola. Thus there are five conic sections. (Somerville's note.)

[12] *Inversely as the square of the distance.* The attraction of one body for another at the distance of two miles is four times less than at the distance of one mile; at three miles it is nine times less than at one; at four miles it is sixteen times less, and so on. That is, the gravitating force decreases in intensity as the squares of the distances increase. (Somerville's note.)

[13] See note 3, *Preliminary Dissertation.*

[14] This was the second of Kepler's three celebrated laws. The first law is, *That the radii vectores of the planets and comets describe areas proportional to the time.* The second law is, *That the orbits or paths of the planets and comets are conic sections, having the sun in one of their foci.* The third law is, *That the squares of the periodic times of the planets are proportional to the cubes of their mean distances from the sun.* (Somerville's note.)

[15] But for the resistance of the air, all bodies would fall to the ground in equal times. In fact, a hundred equal particles of matter at equal distances from the surface of the earth would fall to the ground in parallel straight lines with equal rapidity, and no change whatever would take place in the circumstances of their descent, if 99 of them were united in one solid mass; for the solid mass and the single particle would touch the ground at the same instant, were it not for the resistance of the air. (Somerville's note.)

satellites, such as our moon, also gravitate to their primaries, or the planets about which they revolve, according to the same law that their primaries do to the sun. Thus, by the law of action and reaction, each body is itself the centre of an attractive force extending indefinitely in space, causing all the mutual disturbances which render the celestial motions so complicated, and their investigation so difficult.

The gravitation of matter directed to a centre, and attracting directly as the mass (or the quantity of matter in a given bulk), and inversely as the square of the distance, does not belong to it when considered in mass only; particle acts on particle according to the same law when at sensible distances from each other. If the sun acted on the centre of the earth, without attracting each of its particles, the tides would be very much greater than they now are and would also, in other respects, be very different. The gravitation of the earth to the sun results from the gravitation of all its particles, which, in their turn, attract the sun in the ratio of their respective masses. There is a reciprocal action likewise between the earth and every particle at its surface. The earth and a feather mutually attract each other in the proportion of the mass of the earth to the mass of the feather. Were this not the case, and were any portion of the earth, however small, to attract another portion, and not be itself attracted, the centre of gravity of the earth would be moved in space by this action, which is impossible.

The forms of the planets result from the reciprocal attraction of their component particles. A detached fluid mass, if at rest, would assume the form of a sphere, from the reciprocal attraction of its particles. But if the mass revolve about an axis, it becomes flattened at the poles and bulges at the equator, in consequence of the motion imparted to each particle by the velocity of rotation; for this tendency to direct onward motion diminishes the gravity of the particles especially at the equator where the movement of rotation is most rapid, and equilibrium can only exist where it is balanced by an increase of gravity. Therefore, as the attractive force is the same on all particles at equal distances from the centre of a sphere, the equatorial particles would recede from the centre, till their increase in number by matter brought down from the poles produces a counterbalancing amount of attraction. Consequently, the sphere would become an oblate or flattened spheroid, and a fluid, partially or entirely covering a solid, as the ocean and atmosphere cover the earth, must assume that form in order to remain in equilibrio. The surface of the sea is, therefore, spheroidal, and the surface of the earth only deviates from that figure where it rises above or sinks below the level of the sea. But the deviation is so small, that it is unimportant when compared with the magnitude of the earth; for the mighty chain of the Andes, and the yet more lofty Himalaya, bear about the same proportion to the earth that a grain of sand does to a globe three feet in diameter. Such is the form of the earth and planets. The compression or flattening at their poles is, however, so small, that even Jupiter, whose rotation is the most rapid, and which is therefore the most elliptical of the planets, may, from his great distance, be regarded as spherical. Although the planets attract each other as if they were spheres, on account of their distances, yet the satellites are near enough to be sensibly affected in their motions by the forms of their primaries. The moon, for example, is so near the earth, that the reciprocal attraction between each of her particles, and each of the particles in the prominent mass at the terrestrial equator, occasions considerable disturbances in the motions of both bodies; for the action of the moon on the protuberant matter at the earth's equator produces a nutation, or nodding motion, in the earth's axis of rotation like that of a spinning-top when it is about to fall, and the reaction of that matter on the moon is the cause of a corresponding nutation in the lunar orbit.[16]

If a sphere at rest in space receive an impulse passing through its centre of gravity, all its parts will move with an equal velocity in a straight line; but, if the impulse does not pass through the centre of gravity, its particles, having unequal velocities, will have a rotatory or revolving motion, at the same time that it is translated or carried forward in space. These motions are independent of one another; so that a contrary impulse, passing through its centre of gravity, will impede its progress, without interfering with its rotation. The sun rotates about an axis, and modern observations show that an impulse in a contrary direction has not been given to its centre of gravity, for he moves in space accompanied by all those bodies which compose the solar system–a circumstance which in no way interferes with their relative motions; for, in consequence of the principle that force is proportional to velocity, the reciprocal attractions of a system remain the same whether its centre of gravity be at rest, or moving uniformly in space. It is computed that, had the earth received its motion from a single impulse, that impulse must have passed through a point about twenty-five miles from its centre.

Since the motions of rotation and translation of the planets are independent of each other, though probably communicated by the same impulse, they form separate subjects of investigation.

Elliptical Orbits

A planet moves in its elliptical orbit with a velocity varying every instant, in consequence of two forces, one tending to the centre of the sun, and the other in the direction of a tangent (*mT*, Fig. 78, Article 407) to its orbit, arising from the primitive impulse given at the time when it was launched into space. Should the onward motion of the planet cease, it would fall to the sun by its gravity. Were the sun not to attract it, the planet would fly off in the tangent. Thus, when the planet is at the point of its orbit furthest from the sun, its velocity gradually decreases till it is overcome by the attraction of the sun which brings it back with such an accelerated motion, that when nearest to the sun it overcomes his attraction, and, shooting past him, gradually decreases in velocity until it arrives at the most distant point, where the sun's attraction again prevails.[17] In this motion the *radii vectores* or imaginary lines joining the centres of the sun and the planets, pass over equal areas or spaces in equal times, as stated in Kepler's first law (see note 14).

The mean distance of a planet from the sun is equal to half the major axis of its orbit (PA, fig. 76, article 392), therefore the periodic time, or time of revolution of the planet round the sun, would be the same whether it moved in a circular or elliptical orbit, since the curves coincide at the extremities of the major axis. If the planet described a circle round the sun, its velocity or speed would be the same at all points in its orbit; whereas when moving in an ellipse, its elliptic or true motion will be continually varying, being either faster or slower than the circular or mean

[16] *Nutation of lunar orbit*. The action of the bulging matter at the earth's equator on the moon occasions a variation in the inclination of the lunar orbit to the plane of the ecliptic. (Somerville's note.)

[17] *Motion in an elliptical orbit*. A planet *m*, (fig. 76, article 392) moves round the sun at *S* in an ellipse P*m*AP, in consequence of two forces, one urging it in the direction of the tangent at *m*, and another pulling it towards the sun in the direction *mS*. Its velocity, which is greatest at P, decreases throughout the arc to A, where it is least, and increases continually as it moves along the arc till it comes to P again. The whole force producing the elliptical motion varies inversely as the square of the distance. (Somerville's note.)

motion at all points except the extremities of the major axis where the curves coincide.[18] As it is necessary to have some fixed point in the heavens from whence to estimate these motions, the vernal equinox (article 360) at a given epoch has been chosen. The equinoctial, which is a great circle traced in the starry heavens by the imaginary extension of the plane of the terrestrial equator, is intersected by the ecliptic, or apparent path of the sun, in two points diametrically opposite to one another, called the vernal and autumnal equinoxes. The vernal equinox is the point through which the sun passes in going from the southern to the northern hemisphere; and the autumnal, that in which he crosses from the northern to the southern. The mean or circular motion of a body, estimated from the vernal equinox, is its mean longitude (article 392); and its elliptical, or true motion, reckoned from that point, is its true longitude (article 392), both being estimated from west to east, the direction in which the bodies move. The difference between the two is called the equation of the centre (article 382), which consequently vanishes at the apsides,[19] or extremities of the major axis, and is at its maximum ninety degrees distant from these points, or in quadratures,[20] where it measures the eccentricity[21] of the orbit; so that the place of the planet in its elliptical orbit is obtained by adding or subtracting the equation of the centre to or from its mean longitude.

The orbits of the principal planets have a very small obliquity or inclination to the plane of the ecliptic in which the earth moves;[22] and, on that account, astronomers refer their motions to this plane at a given epoch as a known and fixed position. The angular distance of a planet from the plane of the ecliptic is its latitude,[23] which is south or north according as the planet is south or north of that plane. When the planet is in the plane of the ecliptic, its latitude is zero; it is then said to be in its nodes.[24] The ascending node is that point in the ecliptic through which the planet passes in going from the southern to the northern hemisphere. The descending node is a corresponding point in the plane of the ecliptic diametrically opposite to the other, through which the planet descends in going from the northern to the southern hemisphere. The longitude and latitude of a planet cannot be obtained by direct observation, but are deduced from observations made at the surface of the earth by a very simple computation. These two quantities, however, will not give the place of a planet in space. Its distance from the sun[25] must also be known; and, for the complete determination of its elliptical motion, the nature and position of its orbit must be ascertained by observation. This depends upon seven quantities, called the elements of the orbit (article 378). These are, the length of the major axis, and the eccentricity, which determine the

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[18] Let AMP, fig. 76., article 392, be the upper half of the circular orbit moving round the sun in the centre, C, and A*m*P the elliptic orbit of a body moving round the sun situated at *S*, one of the foci of the ellipse. In both cases the periodic time of the planet would be the same, but whereas when describing the circle, its motion would be uniform in all parts, its speed at different points of the ellipse would be continually varying as explained in the previous note.

[19] *Apsides*. The points P and A, fig. 76, article 392, at the extremities of the major axis of an orbit. P is called the *perihelion*; and the point A the *aphelion*. (Somerville's note.)

[20] *Quadratures*. A celestial body is said to be in quadrature when it is 90 degrees distant from the sun. (Somerville note.)

[21] *Eccentricity*. Deviation from circular form. In fig. 76, article 392, C*S* is the eccentricity of the orbit P*m*AP. (Somerville note.)

[22] Angle PN*p*, fig. 77, article 397.

[23] Angle *mSp*, fig. 77, article 397.

[24] *Nodes*. The two points *n* and N, fig. 77, article 397, in which the orbit of a planet or comet intersects the plane of the ecliptic. The ascending node N is the point through which the body passes in rising above the plane of the ecliptic, and the descending node *n* is the point in which the body sinks below it. (Somerville's note.)

[25] *Distance from the sun*. *Sm* in fig. 77, article 397. When the three quantities: the latitude, the longitude, and the distance from the sun are known, the place of the planet is determined in space. (Somerville's note.)

form of the orbit; the longitude of the planet when at its least distance from the sun, called the longitude of the perihelion; the inclination of the orbit to the plane of the ecliptic, and the longitude of its ascending node: these give the position of the orbit in space; but the periodic time, and the longitude of the planet at a given instant, called the longitude of the epoch, are necessary for finding the place of the body in its orbit at all times. A perfect knowledge of these seven elements is requisite for ascertaining all the circumstances of undisturbed elliptical motion. By such means it is found that the path of the planets, when their mutual disturbances are omitted, are ellipses nearly approaching to circles, whose planes, slightly inclined to the ecliptic, cut it in straight lines, passing through the centre of the sun. The orbits of the recently discovered planets[26] deviate more from the ecliptic than those of the ancient planets: that of Pallas, for instance, has an inclination of $34^\circ\ 1'\ 31''$ to it; on which account it is more difficult to determine their motions.

Were the planets attracted by the sun only, they would always move in ellipses, invariable in form and position and because his action is proportional to his mass, which is much larger than that of all the planets put together, the elliptical is the nearest approximation to their true motions. The true motions of the planets are extremely complicated, in consequence of their mutual attraction, so that they do not move in any known or symmetrical curve, but in paths now approaching to, now receding from, the elliptical form; and their radii vectores do not describe areas or spaces exactly proportional to the time, so that the areas become a test of the disturbing forces.

To determine the motion of each body, when disturbed by all the rest, is beyond the power of analysis. It is therefore necessary to estimate the disturbing action of one planet at a time, whence the celebrated problem of the three bodies, originally applied to the moon, the earth, and the sun—namely, the masses being given of three bodies projected from three given points, with velocities given both in quantity and direction; and supposing the bodies to gravitate to one another with forces that are directly as their masses, and inversely as the square of the distances, to find the lines described by these bodies, and their positions at any given instant; or, in other words, to determine the path of a celestial body when attracted by a second body, and disturbed in its motion round the second body by a third—a problem equally applicable to planets, satellites, and comets.

By this problem the motions of translation of the celestial bodies are determined. It is an extremely difficult one, and would be infinitely more so if the disturbing action were not very small when compared with the central force; that is, if the action of the planets on one another were not very small when compared with that of the sun. As the disturbing influence of each body may be found separately, it is assumed that the action of the whole system, in disturbing any one planet, is equal to the sum of all the particular disturbances it experiences, on the general mechanical principle, that the sum of any number of small oscillations is nearly equal to their simultaneous and joint effect.

Perturbations of the Planets

The planets are subject to disturbances of two kinds, both resulting from the constant operation of their reciprocal attraction: one kind depending upon their positions with regard to

[26] See note 9, *Preliminary Dissertation.*

each other, begins from zero, increases to a maximum, decreases, and becomes zero when the planets return to the same relative positions. In consequence of these, the disturbed planet is sometimes drawn away from the sun, sometimes brought nearer to him: sometimes it is accelerated in its motion, and sometimes retarded. At one time it is drawn above the plane of its orbit, at another time below it, according to the position of the disturbing body. All such changes, being accomplished in short periods, some in a few months, others in years, or in hundreds of years, are denominated *periodic inequalities*. The inequalities of the other kind, though occasioned likewise by the disturbing energy of the planets, are entirely independent of their relative positions. They depend upon the relative positions of the orbits alone, whose forms and places in space are altered by very minute quantities, in immense periods of time, and are therefore called *secular inequalities*.

The *periodical perturbations* are compensated when the bodies return to the same relative positions with regard to one another and to the sun: the *secular inequalities* are compensated when the orbits return to the same positions relatively to one another and to the plane of the ecliptic.

Planetary motion, including both these kinds of disturbance, may be represented by a body revolving in an ellipse, and making small and transient deviations, now on one side of its path, and now on the other, whilst the ellipse itself is slowly, but perpetually. changing both in form and position.

The periodic inequalities are merely transient deviations of a planet from its path, the most remarkable of which only lasts about 918 years; but in consequence of the secular disturbances, the apsides, or extremities of the major axes of all the orbits, have a direct but variable motion in space, excepting those of the orbit of Venus, which are retrograde[27] and the lines of the nodes move with a variable velocity in a contrary direction. Besides these, the inclination and eccentricity of every orbit are in a state of perpetual but slow change. These effects result from the disturbing action of all the planets on each. But, as it is only necessary to estimate the disturbing influence of one body at a time, what follows may convey some idea of the manner in which one planet disturbs the elliptical motion of another.

Suppose two planets moving in ellipses round the sun; if one of them attracted the other and the sun with equal intensity, and in parallel directions, or such as could never meet however much prolonged, it would have no effect in disturbing the elliptical motion. The inequality of this attraction is the sole cause of perturbation, and the difference between the disturbing planet's action on the sun and on the disturbed planet constitutes the disturbing force, which consequently varies in intensity and direction with every change in the relative positions of the three bodies. Although both the sun and planet are under the influence of the disturbing force, the motion of the disturbed planet is referred to the centre of the sun as a fixed point, for convenience. The whole force which disturbs a planet is equivalent to three partial forces. One of these acts on the disturbed planet, in the direction of a tangent to its orbit, and is called the *tangential force*: it occasions secular inequalities in the form and position of the orbit in its own plane, and is the sole cause of the periodical perturbations in the planet's longitude. Another acts upon the same body in the direction of its *radius vector*, that is, in the line joining the centres of the sun and planet, and is called the *radial force*: it produces periodical changes in the distance of the planet

[27] *Retrograde*. Going backwards, as from east to west, contrary to the motion of the planets. This apparent anomaly in the case of Venus is caused by the combined action of the earth and Mercury, by which the apsides of Venus are made to recede more rapidly than the joint action of all the other planets can cause them to advance. (Somerville's note.)

from the sun, and affects the form and position of the orbit in its own plane. The third, which may be called the *perpendicular force*, acts at right angles to the plane of the orbit, occasions the periodic inequalities in the planet's latitude, and affects the position of the orbit with regard to the plane of the ecliptic.

It has been observed, that the radius vector of a planet, moving in a perfectly elliptical orbit, passes over equal spaces or areas in equal times; a circumstance which is independent of the law of the force, and would be the same whether it varied inversely as the square of the distance, or not, provided only that it be directed to the centre of the sun. Hence the tangential force, not being directed to the centre, occasions an unequable description of areas, or, what is the same thing, it disturbs the motion of the planet in longitude. The tangential force sometimes accelerates the planet's motion, sometimes retards it, and occasionally has no effect at all. Were the orbits of both planets circular, a complete compensation would take place at each revolution of the two planets, because the arcs in which the accelerations and retardations take place would be symmetrical on each side of the disturbing force. For it is clear, that if the motion be accelerated through a certain space, and then retarded through as much, the motion at the end of the time will be the same as if no change had taken place. But, as the orbits of the planets are ellipses, this symmetry does not hold: for, as the planet moves unequably in its orbit, it is in some positions more directly, and for a longer time, under the influence of the disturbing force than in others. And, although multitudes of variations do compensate each other in short periods, there are others, depending on peculiar relations among the periodic times of the planets, which do not compensate each other till after one, or even till after many revolutions of both bodies. A periodical inequality of this kind in the motions of Jupiter and Saturn has a period of no less than 918 years.

The radial force, or that part of the disturbing force which acts in the direction of the line joining the centres of the sun and disturbed planet, has no effect on the areas, but is the cause of periodical changes of small extent in the distance of the planet from the sun. It has already been shown, that the force producing perfectly elliptical motion varies inversely as the square of the distance, and that a force following any other law would cause the body to move in a curve of a very different kind. Now, the radial disturbing force varies directly as the distance; and, as it sometimes combines with and increases the intensity of the sun's attraction for the disturbed body, and at other times opposes and consequently diminishes it, in both cases it causes the sun's attraction to deviate from the exact law of gravity, and the whole action of this compound central force on the disturbed body is either greater or less than what is requisite for perfectly elliptical motion. When greater, the curvature of the disturbed planet's path, on leaving its perihelion (article 316), or point nearest the sun, is greater than it would be in the ellipse, which brings the planet to its aphelion (article 316), or point farthest from the sun, before it has passed through 180°, as it would do if undisturbed. So that in this ease the apsides, or extremities of the major axis, advance in space. When the central force is less than the law of gravity requires, the curvature of the planet's path is less than the curvature of the ellipse. So that the planet, on leaving its perihelion, would pass through more than 180° before arriving at its aphelion, which causes the apsides to recede in space. Cases both of advance and recess occur during a revolution of the two planets; but those in which the apsides advance preponderate. This, however, is not the full amount of the motion of the apsides; part arises also from the tangential force, which alternately accelerates and retards the velocity of the disturbed planet. An increase in the planet's tangential velocity diminishes the curvature of its orbit, and is equivalent to a decrease of central force. On the contrary, a decrease of the tangential velocity, which increases the curvature of the orbit, is equivalent to an increase of central force. These fluctuations, owing to the tangential force, occasion an alternate recess and advance of the apsides, after the manner already explained. An uncompensated portion of the direct motion, arising from this cause, conspires with that already impressed by the radial force, and in some cases even nearly doubles the direct motion of these points. The motion of the apsides may be represented by supposing a planet to move in an ellipse, while the ellipse itself is slowly revolving about the sun in the same plane. This motion of the major axis, which is direct in all the orbits except that of the planet Venus is irregular, and so slow that it requires more than 109,880 years for the major axis of the earth's orbit to accomplish a sidereal revolution, that is, to return to its original position among the stars; and 20,984 years to complete its tropical revolution, or to return to the same equinox. The difference between these two periods arises from a retrograde motion in the equinoctial point (see article 360), which meets the advancing axis before it has completed its revolution with regard to the stars. The major axis of Jupiter's orbit requires no less than 200,610 years to perform its sidereal revolution, and 22,748 years to accomplish its tropical revolution from the disturbing action of Saturn alone.

A variation in the eccentricity of the disturbed planet's orbit is an immediate consequence of the deviation from elliptical curvature, caused by the action of the disturbing force. When the path of the body, in proceeding from its perihelion to its aphelion, is more curved than it ought to be from the effect of the disturbing forces, it falls within the elliptical orbit, the eccentricity is diminished, and the orbit becomes more nearly circular; when that curvature is less than it ought to be, the path of the planet falls without its elliptical orbit, and the eccentricity is increased; during these changes, the length of the major axis is not altered, the orbit only bulges out, or becomes more flat. Thus the variation in the eccentricity arises from the same cause that occasions the motion of the apsides. There is an inseparable connection between these two elements: they vary simultaneously, and have the same period; so that whilst the major axis revolves in an immense period of time, the eccentricity increases and decreases by very small quantities, and at length returns to its original magnitude at each revolution of the apsides. The terrestrial eccentricity is decreasing at the rate of about 40 miles annually; and if it were to decrease equably it would be 39,861 years before the earth's orbit became a circle. M. Leverrier[28] has, however, proved that the diminution will not continue beyond 23,980 years, from the year AD 1800; after that time the eccentricity will begin to increase. The mutual action of Jupiter and Saturn occasions variations in the eccentricity of the orbits of both these planets, the greatest eccentricity of Jupiter's orbit corresponding to the least of Saturn's. The period in which these vicissitudes are accomplished is 70,414 years, estimating the action of these two planets alone; but, if the action of all the planets were estimated, the cycle would extend to millions of years.

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[28] Leverrier, Urbain Jean Joseph, (1811-1877), astronomer, born in St. Lô, France. Leverrier predicted the existence and location in the heavens of an undiscovered planet based on the perturbations in the motions of the planets. This deduction occurred eight months after that of J. C. Adams (1819-1892) who in 1845 was inspired to calculate the location of the undiscovered planet based upon a prediction by Mary Somerville written in the 6th edition of her *On the Connexion of the Physical Sciences* published in 1842. Neptune was actually discovered at that location by Galle in Germany on September 23, 1846 within a few days of Leverrier's calculations and at Leverrier's request (see also note 39 below, note 48, *Bk. I, Foreword*, and note 38, *Bk. II, Chap. XIV*). Leverrier focussed much of his attention on the planet Mercury and compiled tables of its motions. He was also intrigued by unusual characteristics of Mercury's motion, phenomena that were finally adequately explained by Einstein's general theory of relativity in 1915.

That part of the disturbing force is now to be considered which acts perpendicularly to the plane of the orbit, causing periodic perturbations in latitude, secular variations in the inclination of the orbit, and a retrograde motion to its nodes on the true plane of the *ecliptic*, or apparent path of the sun round the earth. This force tends to pull the disturbed body above, or push it below, the plane of its orbit, according to the relative positions of the two planets with regard to the sun, considered to be fixed. By this action, it sometimes makes the plane of the orbit of the disturbed body tend to coincide with the plane of the ecliptic, and sometimes increases its inclination to that plane. In consequence of which, its nodes alternately recede or advance on the ecliptic. When the disturbing planet is in the line of the disturbed planet's nodes, it neither affects these points, the latitude, nor the inclination, because both planets are then in the same plane. When it is at right angles to the line of the nodes, and the orbit symmetrical on each side of the disturbing force, the average motion of these points, after a revolution of the disturbed body, is retrograde, and comparatively rapid: but, when the disturbing planet is so situated that the orbit of the disturbed planet is not symmetrical on each side of the disturbing force, which is most frequently the case, every possible variety of action takes place. Consequently, the nodes are perpetually advancing or receding with unequal velocity; but, as a compensation is not effected, their motion is, on the whole, retrograde.

With regard to the variations in the inclination, it is clear, that, when the orbit is symmetrical on each side of the disturbing force, all its variations are compensated after a revolution of the disturbed body, and are merely periodical perturbations in the planet's latitude; and no secular change is induced in the inclination of the orbit. When, on the contrary, that orbit is not symmetrical on each side of the disturbing force, although many of the variations in latitude are transient or periodical, still, after a complete revolution of the disturbed body, a portion remains uncompensated, which forms a secular change in the inclination of the orbit to the plane of the ecliptic. It is true, part of this secular change in the inclination is compensated by the revolution of the disturbing body, whose motion has not hitherto been taken into the account, so that perturbation compensates perturbation; but still a comparatively permanent change is effected in the inclination, which is not compensated till the nodes have accomplished a complete revolution.

The changes in the inclination are extremely minute, compared with the motion of the nodes, and there is the same kind of inseparable connection between their secular changes that there is between the variation of the eccentricity and the motion of the major axis. The nodes and inclinations vary simultaneously; their periods are the same, and very great. The nodes of Jupiter's orbit, from the action of Saturn alone, require 36,261 years to accomplish even a tropical revolution. In what precedes, the influence of only one disturbing body has been considered; but, when the action and reaction of the whole system are taken into account, every planet is acted upon, and does itself act, in this manner, on all the others; and the joint effect keeps the inclinations and eccentricities in a state of perpetual variation. It makes the major axis of all the orbits continually revolve, and causes, on an average, a retrograde motion of the nodes of each orbit upon every other. The ecliptic itself is in motion from the mutual action of the earth and planets, so that the whole is a compound phenomenon of great complexity, extending through unknown ages. At the present time the inclinations of all the orbits are decreasing but so slowly, that the inclination of Jupiter's orbit is only about six minutes less than it was in the age of Ptolemy.[29]

[29] See note 15, *Preliminary Dissertation.*

But, in the midst of all these vicissitudes, the length of the major axes and the mean motions of the planets remain permanently independent of secular changes. They are so connected by Kepler's law of the squares of the periodic times being proportional to the cubes of the mean distances of the planets from the sun, that one cannot vary without affecting the other. And it is proved, that any variations which do take place are transient, and depend only on the relative positions of the bodies.

It is true that, according to theory, the radial disturbing force should permanently alter the dimensions of all the orbits, and the periodic times of all the planets, to a certain degree. For example, the masses of all the planets revolving within the orbit of any one, such as Mars, by adding to the interior mass, increase the attracting force of the sun, which, therefore, must contract the dimensions of the orbit of that planet, and diminish its periodic time; whilst the planets exterior to Mars' orbit must have the contrary effect. But the mass of the whole of the planets and satellites taken together is so small, when compared with that of the sun, that these effects are quite insensible, and could only have been discovered by theory. And, as it is certain that the length of the major axes and the mean motions are not permanently changed by any other power whatever, it may be concluded that they are invariable.

With the exception of these two elements, it appears that all the bodies are in motion, and every orbit in a state of perpetual change. Minute as these changes are, they might be supposed to accumulate in the course of ages sufficiently to derange the whole order of nature, to alter the relative positions of the planets, to put an end to the vicissitudes of the seasons, and to bring about collisions which would involve our whole system, now so harmonious, in chaotic confusion. It is natural to inquire, what proof exists that nature will be preserved from such a catastrophe? Nothing can be known from observation, since the existence of the human race has occupied comparatively but a moment in duration, while these vicissitudes embrace myriads of ages. The proof is simple and conclusive. All the variations of the solar system, secular as well as periodic, are expressed analytically by the sines and cosines of circular arcs which increase with the time; and, as a sine or cosine can never exceed the radius, but must oscillate between zero and unity, however much the time may increase, it follows that when the variations have accumulated to a maximum by slow changes in however long a time, they decrease, by the same slow degrees, till they arrive at their smallest value, again to begin a new course; thus for ever oscillating about a mean value. This circumstance, however, would be insufficient, were it not for the small eccentricities of the planetary orbits, their minute inclinations to the plane of the ecliptic, and the revolutions of all the bodies, as well planets as satellites, in the same direction. These secure the perpetual stability of the solar system.[30] However, at the time that the stability was proved by Lagrange and Laplace, the telescopic planets between Mars and Jupiter had not been discovered; but Lagrange, having investigated the subject under a very general point of view, showed that, if a planetary system be composed of very unequal masses, the whole of the

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[30] The small eccentricities and inclinations of the planetary orbits, and the revolutions of all the bodies in the same direction, were proved by Euler (see note 6, *Book I, Chapter II*), Lagrange (see note 16, *Preliminary Dissertation*), and Laplace (see note 4, *Introduction*), to be conditions necessary for the stability of the solar system. Subsequently, however, the periodicity of the terms of the series expressing the perturbations was supposed to be sufficient *alone*, but M. Poisson (see note 1, *Book I, Chapter 6*) has shown that to be a mistake; that these three conditions are requisite for the necessary convergence of the series, and that therefore the stability of the system depends on them *conjointly* with the periodicity of the sines and cosines of each term. The author (i.e. Somerville) is aware that this note can only be intelligible to the analyst, but she is desirous of correcting an error, and the more so as the conditions of stability afford one of the most striking instances of design in the original construction of our system, and of the foresight and supreme wisdom of the Divine Architect. (Somerville's note.)

larger would maintain an unalterable stability with regard to the form and position of their orbits, while the orbits of the lesser might undergo unlimited changes. M. Leverrier has applied this to the solar system, and has found that the orbits of all the larger planets will for ever maintain an unalterable stability in form and position; for, though liable to mutations of very long periods, they return again exactly to what they originally were, oscillating between very narrow limits; but he found a zone of instability between the orbit of Mars, and twice the mean distance of the earth from the sun,[31] or between 1.5 and 2.00; therefore the position and form of the orbits of such of the telescopic planets as revolve within that zone will be subject to unlimited variations. But the orbits of those more remote from the sun than Flora,[32] or beyond 2.20, will be stable, so that their eccentricities and inclinations must always have been, and will always remain, very great, since they must have depended upon the primitive conditions that prevailed when these planetary atoms were launched into space. The telescopic planets,[33] numerous as they are, 153 having been discovered up to the date of Jan. 1876,[34] have been shown by Leverrier's calculations, completed in 1875, to have no influence upon the motions of Jupiter and scarcely any upon those of Mars. This result was to be expected, for Jupiter has a diameter of 84,846 miles,[35] while that of Pallas, his nearest neighbour, is not more than 171 miles.[36] The diameter of Mars, on the other side of the small planets, is 4,363 miles,[37] and that of the earth 7,920 miles,[38] so that the telescopic group is too minute to disturb the others. M. Leverrier found another zone of instability between Venus and the sun, on the border of which Mercury is revolving the inclination of whose orbit to the plane of the ecliptic is about 70°, which is more than that of any of the large planets. Neptune's orbit is, no doubt, as stable as that of any other of the large planets, as the inclination is very small, but he will have periodical variations of very long duration from the reciprocal attraction between him and Uranus, one especially of an enormous duration, similar to those of Jupiter and Saturn, and, like them, depending on the time of his revolution round the sun being nearly twice as long as that of Saturn. Mr. Adams[39] has computed that Neptune produces a periodical perturbation in the motion of Uranus, whose duration is about 6,800 years.

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[31] The mean distance of the earth from the sun is 91,600,000 miles, but to avoid the inconvenience of large numbers, it is assumed to be the unit of distance; hence the mean distance of Mars is 1.52369, or 1.5 nearly, that of the earth being = 1. (Somerville's note.)

[32] Flora is an asteroid discovered in 1847 by J. R. Hind in London. It has a diameter of about 141 km. Its distance from the Sun varies between 1.86 and 2.55 AU. The asteroid has a period of 3.27 years with an orbit inclined at 5.9° to the ecliptic. *Royal Astronomical Society of New Zealand.*

[33] *telescopic planets.* A term Somerville uses to refer to the asteroids.

[34] Several hundred thousand asteroids are now known with 26 of those larger than 200 km in diameter.

[35] *diameter of Jupiter.* The modern value is 88,850 mi. (142,984 km.)

[36] Stone's *Astronomical Monthly Notices*, vol. xxvii. p. 302. (Somerville's note.)

[37] *diameter of Mars.* Modern value is 4,222 mi. (6,794 km.)

[38] *diameter of Earth.* Modern value is 7,926 mi. (12,756 km.)

[39] Adams, John Couch, (1819-1892), astronomer, born in Laneast, England. Adams mathematically deduced the existence of the planet Neptune based upon the writings of Mary Somerville (see *Foreword to the Second Edition*). Adams gave the Director of the Cambridge Observatory precise data on the (still unseen) planet's location in September, 1845. Adams' calculations were done eight months before French astronomer Leverrier (1811-1877) who performed similar calculations independently. Leverrier then requested a search by the German astronomer Galle who located the planet at the Berlin Observatory a few days later on September 23, 1846. (see also note 28 above, note 48, *Bk. I, Foreword*, and note 38, *Bk. II, Chap. XIV*.)