BOOK I - DYNAMICS

—

FOREWORD[1]

The Figure of the Earth

THE theoretical investigation of the figure of the earth and planets is so complicated, that neither the geometry of Newton[2] nor the refined analysis of Laplace[3] has attained more than an approximation. The solution of that difficult problem was greatly advanced by the late Mr. Ivory.[4] The investigation has been conducted by successive steps, beginning with a simple case, and then proceeding to the more difficult. But, in all, the forces which occasion the revolutions of the earth and planets are omitted, because, by acting equally on all the particles, they do not disturb their mutual relations. A fluid mass of uniform density, whose particles mutually gravitate to one another, will assume the form of a sphere when at rest. But, if the sphere begins to revolve, every particle will describe a circle having its centre in the axis of revolution. The planes of all these circles will be parallel to one another and perpendicular to the axis, and the particles will have a tendency to fly from that axis in consequence of the centrifugal force arising from the velocity of rotation. The force of gravity is everywhere perpendicular to the surface, and tends to the interior of the fluid mass; whereas the centrifugal force, acts perpendicularly to the axis of rotation, and is directed to the exterior. And, as its intensity diminishes with the distance from the axis of rotation, it decreases from the equator to the poles, where it ceases. Now it is clear that these two forces are in direct opposition to each other in the equator alone, and that gravity is there diminished by the whole effect of the centrifugal force, whereas, in every other part of the fluid, the centrifugal force is resolved into two parts, one of which, being perpendicular to the surface, diminishes the force of gravity; but the other, being at a tangent to the surface, urges the particles toward the equator, where they accumulate till their numbers compensate the diminution of gravity, which makes the mass bulge at the equator, and become flattened at the poles. It appears, then, that the influence of the centrifugal force is most powerful at the equator, not only because it is actually greater there than elsewhere, but because its whole effect is employed in diminishing gravity, whereas, in every other point of the fluid mass, it is only a part that is so employed. For both these reasons, it gradually decreases toward the poles, where it ceases. On the contrary, gravity is least at the equator, because the particles are farther from the centre of the mass, and increases toward the poles, where it is greatest. It is evident, therefore, that, as the centrifugal force is much less than gravity—gravitation, which is the difference between the two, is least at the equator, and continually increases towards the poles, where it is a maximum. On these principles Sir Isaac Newton proved that a homogeneous fluid

---

[1] The material in this and the subsequent forewords to Books II, III and IV is drawn from the 10th and last edition of Mary Somerville's *On the Connexion of the Physical Sciences*, (corrected and revised by Arabella B. Buckley), p. 4-106, London : John Murray, 1877.

[2] See note 1, *Preliminary Dissertation.*

[3] See note 18, *Preliminary Dissertation.*

[4] Ivory, James, Sir, (1765-1842), a mathematician who, like Somerville, was an exponent of the French analysis. Ivory did extensive work on the figure of the earth. He also wrote a critical commentary on Laplace's *Mécanique céleste* that Laplace praised. Ivory's mathematical research focussed on the gravitational attraction of ellipsoids, cometary orbits, and atmospheric refraction.

mass in rotation assumes the form of an ellipsoid of revolution. whose compression is $\frac{1}{230}$. Such, however, cannot be the form of the earth, because the strata increase in density towards the centre. The lunar inequalities also prove the earth to be so constructed: It was requisite, therefore, to consider the fluid mass to be of variable density, in which case the compression or flattening would be less than in the case of the homogeneous fluid. Moreover the compression is still less when the mass is considered to be, as it probably is, a solid nucleus decreasing regularly in density from the centre to the surface, and partially covered by the ocean, because the solid parts, by their cohesion, nearly destroy that part of the centrifugal force which gives the particles a tendency to accumulate at the equator, though not altogether; otherwise the sea, by the superior mobility of its particles, would flow towards the equator, and leave the poles dry. Besides, it is well known that the continents at the equator are more elevated than they are in higher latitudes. It is also necessary for the equilibrium of the ocean that its density should be less than the mean density of the earth, otherwise the continents would be perpetually liable to inundations from storms and other causes. Taking all these elements into consideration, it appears from theory, that a horizontal line passing round the earth through both poles must be nearly an ellipse, having its major axis in the plane of the equator, and its minor axis coincident with the axis of the earth's rotation, and Clairaut (see note 14, *Bk. III, Chap II*) and others have determined by mathematical analysis that the equatorial diameter of the spheroid exceeds the polar by $\frac{1}{1152}^{\text{th}}$ of its length, agreeing completely with the fraction deduced from the inequalities of the motion of the moon, and also with the results of actual measurement. It is easy to show in a spheroid whose strata are elliptical, that the increase in the length of the radii, the decrease of gravitation, and the increase in the length of the arcs of the meridian, corresponding to angles of one degree, from the poles to the equator, are all proportional to the square of the cosine of the latitude. These quantities are so connected with the ellipticity of the spheroid, that the total increase in the length of the radii is equal to the compression or flattening, and the total diminution in the length of the arcs is equal to the compression, multiplied by three, times the length of an arc of one degree at the equator. Hence, by measuring the meridian curvature of the earth, the compression, and consequently its figure, become known. This, indeed, is assuming the earth to be an ellipsoid of revolution; but the actual measurement of the globe, will show how far it corresponds with that solid in figure and constitution.

The courses of the great rivers, which are in general navigable to a considerable extent, prove that the curvature of the land differs but little from that of the ocean; and, as the heights of the mountains and continents are inconsiderable when compared with the magnitude of the earth, its figure is understood to be determined by a surface at every point perpendicular to the direction of gravitation, or of the plumb-line, and is the same which the sea would have if it were continued all round the earth beneath the continents. Such is the figure that has been measured in the following manner:–

A terrestrial meridian is a line passing through both poles, all the points of which have their noon contemporaneously. Were the lengths and curvatures of different meridians known, the figure of the earth might be determined. But the length of one degree is sufficient to give the figure of the earth, if it be measured on different meridians, and in a variety of latitudes. For, if the earth were a sphere, all degrees would be of the same length; but, if not, the lengths of the degrees would be greater, exactly in proportion as the curvature is less. It may appear at first sight a paradox to assert that the lengths of the radii of a spheroidal body increase from the poles to the equator and at the same time to state that the length of the degrees marked on the surface increases from the equator to the poles. This apparent contradiction has proved a stumbling-block to many students, but the solution lies in the fact that in the first statement the measurement is supposed to be made from the centre of the spheroid, whereas that point is no longer the centre of equilibrium for all parts of the surface, as it would be in a true sphere. Consequently a plumb-line perpendicular to the surface of the globe will not point to the earth's centre at all, but to a different centre for every point on a meridian of the spheroid, and an angle of one degree of latitude at the poles having a much longer radius, will also have a longer arc than an angle of one degree at the equator. A comparison of the length of a degree in different parts of the earth's surface will therefore determine its size and form.

An arc of the meridian may be measured by determining the latitude of its extreme points by astronomical observations, and then measuring the distance between them in feet or fathoms. The distance thus determined on the surface of the earth, divided by the degrees and parts of a degree contained in the difference of the latitudes, will give the exact length of one degree, the difference of the latitudes being the angle contained between the verticals at the extremities of the arc. This would be easily accomplished were the distance unobstructed and on a level with the sea. But, on account of the innumerable obstacles on the surface of the earth, it is necessary to connect the extreme points of the arc by a series of triangles the sides and angles of which are either measured or computed, so that the length of the arc is ascertained with much laborious calculation. In consequence of the irregularities of the surface each triangle is in a different plane. They must therefore be reduced by computation to what they would have been had they been measured on the surface of the sea. And, as the earth may in this case be esteemed spherical, they require a correction to reduce them to spherical triangles. The officers who conducted the trigonometrical survey in measuring 500 feet of a base in Ireland twice over, found that the difference in the two measurements did not amount to the 800th part of an inch; and in the General Survey of Great Britain, five bases were measured from 5 to 7 miles long and some of them 400 miles apart, yet, when connected by series of triangles, the measured and computed lengths did not differ by more than 3 inches, a degree of accuracy which shows with what care these operations are conducted.

Arcs of the meridian have been measured in a variety of latitudes in both hemispheres, as well as arcs perpendicular to the meridian. From these measurements it appears that the length of the degrees increases from the equator to the poles, nearly in proportion to the square of the sine of the latitude. Consequently the convexity of the earth diminishes from the equator to the poles.

Were the earth a homogeneous ellipsoid of revolution, the meridians would be ellipses whose lesser axes would coincide with the axis of rotation, and all the degrees measured between the pole and the equator would give the same compression when combined two and two. That, however, is far from being the case. Scarcely any of the measurements give exactly the same results, chiefly on account of local attractions, which cause the plumb-line to deviate from the vertical. The vicinity of mountains produces that effect. One of the most remarkable anomalies of this kind has been observed in certain localities of northern Italy, where the action of some dense subterraneous matter causes the plumb-line to deviate seven or eight times more than it did from the attraction of Chimborazo,[5] in the observations of Bouguer (see note 51, *Bk. II, Chap. XIV*), while measuring a degree of the meridian at the equator. In consequence of this local attraction, the degrees of the meridian in that part of Italy seem to increase towards the equator

[5] Chimborazo, Ecuador Location: 1.46S 78.82W. Chimborazo is located in the Inter-Andean Graben, a north-northeast trend structural depression that separates the Western and Eastern Cordillera of the Andes in Ecuador.

through a small space, instead of decreasing, as if the earth was drawn out at the poles, instead of being flattened.

Many other discrepancies occur, but from the mean of the five principal measurements of arcs in Peru, India, France, England, and Lapland, Mr. Ivory deduced that the figure which most nearly follows this law is an ellipsoid of revolution whose equatorial radius would be 3902.824 miles, and the polar radius, 3949.585 miles. The difference, or 13.239 miles, divided by the equatorial radius, would be $\frac{1}{299}$ nearly. This fraction is called the compression of the earth, and does not differ much from that given by the lunar inequalities.[6] Since the preceding quantities were determined, arcs of the meridian have been measured in various parts of the globe, of which the most extensive are the Russian arc of $25^{\circ}20'$ between the Glacial Sea and the Danube, conducted under the superintendence of M. Struve,[7] and the Indian arc extended to $21^{\circ}\ 21'$ by Colonel Everest.[8] All these measurements executed in various parts of the world were compared by Captain A. R. Clarke in an elaborate memoir to the Astronomical Society in 1860, in which he arrived at the following result: 'The earth is not exactly an ellipsoid of revolution. The equator itself is slightly elliptic, the longer and shorter diameters being respectively 41,852,864 and 41,843,096 feet. The ellipticity of the equatorial circumference is therefore $\frac{1}{4283}$ and the excess of its longer over its shorter diameter about two miles. The vertices of the longer diameter are situated in longitudes $14^{\circ}\ 23'$ and $194^{\circ}\ 23'$ E of Greenwich, and of its shorter in $104^{\circ}\ 23'$ and $284^{\circ}\ 23'$ E. The polar axis of the earth is 41,707,796 feet in length, and consequently the most elliptic meridian (that of long. $14^{\circ}\ 23'$ and $194^{\circ}\ 23'$) has for its ellipticity $\frac{1}{287.5}$ and the least–(that of long. $104^{\circ}\ 23'$ and $284^{\circ}\ 23'$) an ellipticity of $\frac{1}{308.3}$.' It appears, therefore, that our globe is not only flattened at the poles, but that the protuberance at the equator is slightly compressed in one direction, so that a line drawn through the centre of the earth from Loango[9] on the West Coast of Africa to the Centre of the Polynesian islands would be two miles longer than a similar line drawn from Sumatra to Equador on the West Coast of South America.

Eratosthenes,[10] who died 194 years before the Christian era, was the first to give an approximate value of the earth's circumference, by the measurement of an arc between Alexandria and Syrene.

There is another method of finding the figure of the earth, totally different from the preceding, solely depending upon the increase of gravitation from the equator to the poles. The force of gravitation at any place is measured by the descent of a heavy body during the first second of its fall. And the intensity of the centrifugal force is measured by the deflection of any point from the tangent in a second. For, since the centrifugal force balances the attraction of the earth, it is an exact measure of the gravitating force. Were the attraction to cease, a body on the surface of the earth would fly off in the tangent by the centrifugal force, instead of bending round in the circle of rotation. Therefore, the deflection of the circle from the tangent in a second measures the intensity of the earth's attraction, and is equal to the versed sine of the arc

---

[6] See Book III.

[7] See note 29, *Preliminary Dissertation*.

[8] Everest, George, Sir, (1790-1866), military engineer, born in Gwernvale, Wales. He worked on the trigonometrical survey of India (1818-43). Mt Everest was renamed in his honour.

[9] Loango, Kingdom of, also called Brama Kingdom, former African state in the basin of the Kouilou and Niari rivers (now largely in southwestern Congo). *Encyclopaedia Britannica*

[10] See note 29, *Preliminary Dissertation*.

described during that time, a quantity easily determined from the known velocity of the earth's rotation. Whence it has been found that at the equator the centrifugal force is equal to the 289th part of gravity. Now it is proved by analysis that, whatever the constitution of the earth and planets may be, if the intensity of gravitation at the equator be taken equal to unity, the sum of the compression of the ellipsoid, and the whole increase of gravitation from the equator to the pole, is equal to five halves of the ratio of the centrifugal force to gravitation at the equator. This quantity with regard to the earth is $\frac{5}{2}$ of $\frac{1}{289}$ or $\frac{1}{115.2}$. Consequently the compression of the earth is equal to $\frac{1}{115.2}$ diminished by the whole increase of gravitation. So that its form will be known, if the whole increase of gravitation from the equator to the pole can be determined by experiment. This has been accomplished by a method founded upon the following considerations:–If the earth were a homogeneous sphere without rotation, its attraction on bodies at its surface would be everywhere the same. If it be elliptical and of variable density the force of gravity, theoretically ought to increase from the equator to the pole, as unity plus a constant quantity multiplied into the square of the sine of the latitude. But for a spheroid in rotation the centrifugal force varies, by the laws of mechanics, as the square of the sine of the latitude, from the equator, where it is greatest, to the pole, where it vanishes. And, as it tends to make bodies fly off the surface, it diminishes the force of gravity by a small quantity. Hence, by gravitation, which is the difference of these two forces, the fall of bodies ought to be accelerated from the equator to the poles proportionally to the square of the sine of the latitude; and the weight of the same body ought to increase in that ratio. This is directly proved by the oscillations of the pendulum, which, in fact, is a falling body; for, if the fall of bodies be accelerated, the oscillations will be more rapid: in order, therefore, that they may always be performed in the same time, the length of the pendulum must be altered. By numerous and careful experiments it is proved that a pendulum, which oscillates 86,400 times in a mean day at the equator, will do the same at every point of the earth's surface, if its length be increased progressively to the pole, as the square of the sine of the latitude.

From the mean of these it appears that the whole decrease of gravitation from the poles to the equator is 0.0051, which, subtracted from $\frac{1}{115.2}$, shows that the compression of the terrestrial spheroid is about $\frac{1}{285.26}$. This value was deduced by the late Mr. Baily,[11] President of the Astronomical Society, who devoted much attention to this subject; at the same time, it may be observed that no two sets of pendulum experiments give the same result, probably from local attractions. The compression obtained by this method does not differ much from that given by the lunar inequalities nor from the arcs in the direction of the meridian, and those perpendicular to it. The near coincidence of these three values, deduced by methods so entirely independent of each other, shows that the mutual tendencies of the centres of the celestial bodies to one another, and the attraction of the earth for bodies at its surface, result from the reciprocal attraction of all their particles. Another proof may be added. The nutation[12] of the earth's axis, and the precession of the equinoxes,[13] are occasioned by the action of the sun and moon on the protuberant matter at the earth's equator. And, although these inequalities do not give the absolute value of the

[11] Baily, Francis, (1774-1844), astronomer, born in Newbury, England. he is known for a phenomenon known as Baily's beads detected during an eclipse of the Sun in 1836. He also calculated the mean density and elliptical shape of the earth by repeating Henry Cavendish's experiments.
[12] see note 5, *Preliminary Dissertation.*
[13] see note 26, *Preliminary Dissertation.*

terrestrial compression, they show that the fraction expressing it is comprised between the limits $\frac{1}{279}$ and $\frac{1}{573}$.

It might be expected that the same compression should result from each, if the different methods of observation could be made without error. This, however, is not the case; for after allowance has been made for every cause of error, such discrepancies are found, both in the degrees of the meridian and in the length of the pendulum, as show that the figure of the earth is very complicated. But they are so small, when compared with the general results, that may be disregarded. The compression deduced from the mean of the whole appears not to differ much from $\frac{1}{300}$; that given by the lunar theory has the advantage of being independent of the irregularities of the earth's surface and of local attractions. The irregularity with which the observed variation in the length of the pendulum follows the law of the square of the sine of the latitude proves the strata to be elliptical, and symmetrically disposed round the centre of gravity of the earth, which affords a strong presumption in favour of its original fluidity. It is remarkable how little influence the sea has on the variation of the lengths of the arcs of the meridian, or on gravitation; neither does it much affect the lunar inequalities, from its density being only about a fifth of the mean density of the earth. For, if the earth were to become fluid after being stripped of the ocean, it would assume the form of an ellipsoid of revolution whose compression is $\frac{1}{304.8}$, which differs very little from that determined by observation, and proves, not only that the density of the ocean is inconsiderable, but that its mean depth is very small. There are, it is true, profound cavities in the bottom of the sea, for recent soundings by the 'Challenger' and other vessels show that the North Pacific is very deep, sometimes exceeding four or five miles over a large portion of its area. The Atlantic Ocean is also more than four miles deep in places, and there is a great central trough extending from near the coast of New York to near the Cape of Good Hope. But the central portion of the principal basin of the North Atlantic is occupied by a plateau, the greater part of which is less than two miles in depth. The South Pacific also, although not thoroughly explored, seems to be decidedly shallower than the North Pacific. On the whole, Dr. Carpenter estimates that the mean depth of the ocean generally may be taken at about two miles.[14] This depth is so insignificant when compared with the size of the earth that immense tracts of land might rise above or sink below the ocean level, as appears really to have been the case, without any great change in the form of the terrestrial spheroid. The variation in the length of the pendulum was first remarked by Richter in 1672, while observing transits of the fixed stars across the meridian at Cayenne,[15] about five degrees north of the equator. He found that his clock lost at the rate of $2^m\ 28^s$ daily, which induced him to determine the length of a pendulum beating seconds in that latitude; and, repeating the experiments on his return to Europe, be found the seconds' pendulum at Paris to be more than the twelfth of an inch longer than that at Cayenne. The form and size of the earth being determined, a standard of measure is furnished with which the dimensions of the solar system may be compared.

The Rotation of the Earth

The rotation of the earth which determines the length of the day, may he regarded as one of the most important elements in the system of the world. It serves as a measure of time, and

[14] *Encyclopaedia Britannica*, 9th edit., article 'Atlantic.' (Somerville's footnote.)
[15] Capital of French Guiana, on Cayenne Island, at the mouth of the Cayenne River.

forms the standard of comparison for the revolutions of the celestial bodies, which, by their proportional increase or decrease, would soon disclose any changes it might sustain. Theory and observation concur in proving that, among the innumerable vicissitudes which prevail throughout creation, the period of the earth's diurnal rotation has remained practically unchanged. The water of rivers, falling from a higher to a lower level, carries with it the velocity due to its revolution with the earth at a greater distance from the centre; it will therefore accelerate, although to an almost infinitesimal extent, the earth's daily rotation. The sum of all these increments of velocity, arising from the descent of all the rivers on the earth's surface, would in time become perceptible, did not nature, by the process of evaporation, raise the waters back to their sources, and thus, by again removing matter to a greater distance from the centre, destroy the velocity generated by its previous approach; so that the descent of rivers does not affect the earth's rotation. The disturbing action of the moon and planets, which has so powerful an effect on the revolution of the earth, has no effect upon its rotation beyond that produced by the friction of the tides, and this is so exceedingly slight as to require thousands of millions of years to produce effects of any magnitude.[16] The constant friction of the trade-winds on the mountains and continents between the tropics does not impede its velocity, which theory even proves to be the same as if the sea, together with the earth, formed one solid mass. But, although these circumstances be insufficient, a variation in the mean temperature would certainly occasion a corresponding change in the velocity of rotation. In the science of dynamics it is a principle in a system of bodies or of particles revolving about a fixed centre, that the momentum or sum of the products of the mass of each into its angular velocity and distance from the centre is a constant quantity, if the system be not deranged by a foreign cause. Now, since the number of particles in the system is the same whatever its temperature may be, when their distances from the centre are diminished, their angular velocity must be increased, in order that the preceding quantity may still remain constant. It follows, then, that, as the primitive momentum of rotation with which the earth was projected into space must necessarily remain the same, the smallest decrease in heat, by contracting the terrestrial spheroid, would accelerate its rotation, and consequently diminish the length of the day [and reciprocally increase the length of the day if the mean temperature were to increase–*ed. note*]. Notwithstanding the constant accession of heat from the sun's rays, geologists have been induced to believe, from the formation of mountain chains and the contorted strata occurring in them, that the mean temperature of the globe is decreasing.

The high temperature of mines, hot springs, and above all the internal fires which have produced, and do still occasion, such devastation on our planet, indicate a gradual augmentation of heat below the surface. The increase of density corresponding to the depth and the form of the spheroid, being what theory assigns to a fluid mass in rotation, concurs to induce the idea that the temperature of the earth was originally so high as to reduce all the substances of which it is composed to a state of fusion or of vapour, and that in the course of ages it has cooled down to its present state; that it is still becoming colder; and that it will continue to do so till the whole mass arrives at the temperature of the medium in which it is placed or rather at a state of equilibrium between this temperature, the cooling power of its own radiation, and the heating effect of the sun's rays.

Previous to the formation of ice at the poles, the ancient lands of northern latitudes might, no doubt, have been capable of producing those tropical plants preserved in the coal-measures, if indeed such plants could flourish without the intense light of a tropical sun. But, even if the decreasing temperature of the earth be sufficient to produce the observed effects, it must be

[16] Stone, *Astronomical Monthly Notices*, vol. xxvii. p. 197. (Somerville's note.)

extremely slow in its operation; for, in consequence of the rotation of the earth being a measure of the periods of the celestial motions, it has been proved that, if the length of the day had decreased by the three-thousandth part of a second since the observations of Hipparchus[17] two thousand years ago, it would have diminished the secular equation of the moon[18] by $44''.4$. It is, therefore, beyond a doubt that the mean temperature of the earth cannot have sensibly varied during that time. If, then, the appearances exhibited by the strata are really owing to a decrease in internal temperature, it either shows the immense periods requisite to produce geological changes, to which two thousand years are as nothing, or that the mean temperature of the earth had arrived at a state of equilibrium before these observations of Hipparchus.

Another cause of decrease of temperature which has been suggested is that of the secular variation of the eccentricity of the earth's orbit,[19] for as Sir John Herschel pointed out in 1835, the total quantity of heat received by the earth from the sun is inversely proportional to the minor axis; or in other words as the minor axis grows longer and the earth's orbit approaches more nearly to a circle we receive less heat from the sun. The utmost difference of heat, however, arising from this cause can never exceed the ratio of 1003 to 1000, and is therefore of very little importance.[20]

Of the decrease in temperature of the northern hemisphere there is abundant evidence in the fossil plants discovered in very high latitudes, which could only have existed in a tropical climate, and which must have near the spot where they are found, from the delicacy of their structure and the perfect state of their preservation. This change of temperature, has again been ascribed to an excess in the duration of spring and summer in the northern hemisphere, in consequence of the eccentricity of the solar ellipse. The length of the season varies with the position of the perihelion of the earth's orbit[21] for two reasons. On account of the eccentricity, small as it is, any line passing through the centre of the sun divides the terrestrial ellipse into two unequal parts, and by the laws of elliptical motion the earth moves through these two portions with unequal velocities.[22] The perihelion always lies in the smaller portion, and there the earth's motion is the most rapid. In the present position of the perihelion, spring and summer north of the equator exceed by about eight days the duration of the same seasons south of it. And 10,500 years ago the southern hemisphere enjoyed the advantage we now possess from the secular variation of the perihelion, Yet Sir John Herschel has shown that by this alternation neither hemisphere receives any excess of light or heat above the other; for, although the earth is nearer to the sun while moving through that part of its orbit in which the perihelion lies than in the other part, and consequently receives a greater quantity of light and heat, yet as it moves faster it is exposed to the heat for a shorter time. In the other part of the orbit, on the contrary, the earth being farther from the sun, receives fewer of his rays; but because its motion is slower, it is exposed to them for a longer time; and, as in both cases the quantity of heat and the angular velocity vary exactly in the same proportion, a perfect compensation takes place in the quantity of heat received from the sun.

Although, however, the mean temperature of the earth as a whole must have remained for ages a constant quantity, yet there is a way in which this unequal division of the heat during

[17] See note 32, *Preliminary Dissertation.*
[18] See *Book III, Chapter I, Article 720.*
[19] See *Book II, Chapter XIV, Article 646.*
[20] Sir J. Herschel, *Trans. of Geol. Society*, 2nd series, vol. iii. (Somerville's note.)
[21] See *Book II, Chapter II, Article 316.*
[22] See *Book II, Chapter II, fig, 63.*

different parts of the year may affect climate. Sir John Herschel pointed out in 1858, that the climates of the southern hemisphere are more extreme than those of the northern on account of the long cold winter endured by the south pole when the earth is in aphelion,[23] the south pole being then turned away from the sun, and the short fierce summer when the earth is in perihelion[24] and the south pole is turned towards the sun. These extremes he showed would become still more marked when the eccentricity of the earth's orbit was greater, as it has been in past ages. Still it seemed that in the course of each year these extremes must compensate each other; but in 1864, Mr. Croll[25] pointed out that such need not necessarily be the case, because the south pole during its long winter would have become covered with immense thicknesses of snow and ice which must be melted before the hot summer sun could warm the earth and raise the general temperature.

So long as water and aqueous vapour remain in their liquid and gaseous state, no difference of temperature in different parts of the earth during successive seasons can produce any cumulative effect, because this very difference causes a circulation throughout the globe which continually tends to bring about equilibrium. But when vapour is changed into snow, and water into ice, a totally different state of things is brought about. Circulation is comparatively stopped, especially if, as Mr. Croll contends, the heat of the succeeding summer is greatly neutralised by the dense fogs which must arise from the condensation of the vapour in contact with the ice covering. Thus the snow and ice are only partially converted into water and vapour, and when the long and cold winter returns an additional accumulation occurs, and thus cold is stored up in each succeeding winter while there is no corresponding storing up of heat.

In this way, when the eccentricity of the earth's orbit was at its maximum, and the extremes of climate greatly increased, that hemisphere which had its winter in aphelion might go on adding to its cap of snow and ice each successive year, till effects might be produced such as would account for the Glacial Period, of which we find so many traces in Europe and America. We have already seen that the conditions would be reversed for each hemisphere every 10,500 years, so that in this way glacial phenomena might be brought about in each hemisphere in succession, while the other hemisphere was enjoying a warm or temperate climate.

But there is another powerful cause which must probably combine with these astronomical changes in order to produce the required effects. Sir Charles Lyell, in his *Principles of Geology* refers the increased cold of the Glacial Period in great part to changes in the position of land and sea, such as we know to have taken place since the earliest geological periods. The loftiest mountains would be represented by a grain of sand on a globe six feet in diameter, and the depth of the ocean by a scratch on its surface. Consequently the gradual elevation of a continent or chain of mountains above the surface of the ocean, or their depression below it, is no very great event compared with the magnitude of the earth and the energy of its subterranean fires, if we take into account the immense periods of time during which these changes must have been in progress, as shown by the successive and various races of extinct beings entombed in the earth's crust. '*Continents, therefore, although permanent for whole geological epochs, shift their positions entirely in the course of ages, and it is not too much to say that every spot which is now dry land has been sea at some former period, and every part of the space now covered by the deepest ocean has been land.*'[26] Now such changes in the disposition of land upon the globe

---

[23] Point A, *Book II, Chapter II, fig, 63*.
[24] Point P, *Book II, Chapter II, fig, 63.*
[25] *Philosophical Magazine*, August, 1864. (Somerville's note.)
[26] *Principles of Geology*, 12th edition, pp. 258, 260. (Somerville's note.)

must affect climate, for variations of temperature are always more intense in the interior of continents than in islands or sea-coasts. An increase of land within the tropics would therefore augment the general heat, and an increase in the temperate and frigid zones would render the cold more severe.

There is at the present time an abnormal quantity of land in polar and circumpolar regions, the proportion of land to sea being as 1 to $2\frac{1}{2}$, whereas in the tropical regions it is only as 1 to 4. Sir C. Lyell,[27] therefore, considered that this might partly account for the amount of ice and snow now accumulated at both poles, and that the milder climates indicated by the fossil remains of some geological formations might be due to a more equable disposition of land and sea; while periods of intense cold, such as the Glacial Period, would be brought about by a still greater preponderance of land towards the poles than now exists.

In the present state of our knowledge it is very difficult to decide how far these different causes of change of climate have aided or counteracted each other, but it is certain that all of them must have had some influence. One more disturbing cause still remains to be mentioned, namely, the variation in the obliquity of the ecliptic,[28] or in the angle of the earth's axis of rotation to the plane of its orbit, which causes the poles to be turned more or less directly towards the sun. At the present time this angle amounts to $23^\circ\ 28'$, but it is decreasing at the rate of $48''$ per century, and will continue to do so for a long time yet to come; after that it will again increase, and will thus continue to oscillate as much as $1^\circ\ 21'$ on one side or the other of a mean position. This range of obliquity is so small that it makes very little difference in the presentation of the polar regions to the sun, and it is therefore of very little importance to climate, but Sir J. Herschel was of the opinion that in the course of millions of years, the deviation might become as great as 3° or 4° on each side of the mean, and if this be so it would be of great assistance to the geologist in helping to account for the tropical Miocene plants found in the polar regions, which must have required not only warmth but light, such as they cannot receive in the present position of our axis of rotation.[29]

It is evident from the marine shells found on the tops of the highest mountains and in almost every part of the globe, that immense continents have been elevated above the ocean, while others have sunk below it. If it were possible for the axis of rotation to alter with reference to the surface of the earth, the seas tending to a new equator would leave some portions of the globe and overwhelm others. Now it is found by the laws of mechanics that in every body, be its form or density what it may, there are at least three axes at right angles to each other, round any one of which, if the solid begins to rotate, it will continue to revolve for ever, provided it be not disturbed by a foreign cause, but that the rotation about any other axis will only be for an instant, and consequently the poles or extremities of the instantaneous axis of rotation would perpetually change their position on the surface of the body. In an ellipsoid of revolution the polar diameter and every diameter in the plane of the equator are the only permanent axes of rotation. Hence, if the ellipsoid were to begin to revolve about any diameter between the pole and the equator, the motion would be so unstable that the axis of rotation and the position of the poles would change every instant. Therefore, as the earth does not differ much from this figure, if it did not turn

---

[27] Lyell, Charles, Sir, (1797-1875), Geologist, born in Kinnordy, Scotland. His famous *Principles of Geology* (1830-3) argued that geological change was brought about by factors that were still at play (uniformitarianism). Lyell communicated frequently with Somerville, Charles Darwin (who praised Lyell lavishly), and Sir John Herschel. Lyell was buried in Westminster Abbey.

[28] Angle P*np*, fig. 81, *Book II, Chapter V, Article 410*.

[29] Lyell, *Principles of Geology*, 12th edition, p. 293. (Somerville's note.)

round one of its principal axes, the position of the poles would change daily; the equator, which is 90° distant, would undergo corresponding variations; and the geographical latitudes of all places, being estimated from the equator, assumed to be fixed, would be perpetually changing. A displacement in the position of the poles of only two hundred miles would be sufficient to produce these effects, and would immediately be detected. But, as the latitudes are found to be invariable, it may be concluded that the terrestrial spheroid must have revolved about the same axis for ages.[30] The earth and planets differ so little from ellipsoids of revolution, that in all probability any libration from one axis to another, produced by the primitive impulse which put them in motion, must have ceased soon after their creation from the friction of the fluids at their surface.

Theory also proves that neither nutation, precession, nor any of the disturbing forces that affect the system, have the smallest influence on the axis of rotation, which maintains a permanent position, if the earth be not disturbed in its rotation by a foreign cause, as the collision of a comet, which might have happened in the immensity of time. But, had that been the case, its effects would still have been perceptible in the variations of the geographical latitudes. If we suppose that such an event had taken place, and that the disturbance had been very great, equilibrium could then only have been restored with regard to a new axis of rotation by the rushing of the seas to the new equator, which they must have continued to do till the surface was everywhere perpendicular to the direction of gravity. But it is probable that such an accumulation of the waters would not be sufficient to restore equilibrium if the derangement had been great, for the mean density of the sea is only about a fifth part of the mean density of the earth, and the mean depth of the Pacific Ocean is supposed not to be more than four or five miles, whereas the equatorial diameter of the earth exceeds the polar diameter by about $26\frac{1}{2}$ miles. Consequently the influence of the sea on the direction of gravity is very small. And, as it thus appears that a great change in the position of the axis is incompatible with the law of equilibrium, the geological phenomena in question must be ascribed to an internal cause. Indeed it is now demonstrated that the strata containing marine diluvia which are in lofty situations must have been formed at the bottom of the ocean, and afterwards upheaved by the action of subterraneous fires. Besides, it is clear, from the mensuration of the arcs of the meridian and the length of the seconds' pendulum, as well as from the lunar theory, that the internal strata and also the external outline of the globe are elliptical, their centres being coincident and their axes identical with that of the surface–a state of things which is incompatible with a subsequent accommodation of the surface to a new and different state of rotation from that which determined the original distribution of the component matter. Thus, amidst the mighty revolutions which have swept innumerable races of organized beings from the earth, which have elevated plains and buried mountains in the ocean, the rotation of the earth and the position of the axes on its surface can have undergone but slight variations.

The question of the nature of the interior of our planet has long been a matter of dispute, both among geologists and astronomers; and it is probable that we shall never arrive at a very definite answer. There can, however, be no doubt that the phenomena of precession and nutation, as they now occur, are almost exactly such as ought to take place if the earth were a continuous solid. Mr. Hopkins, therefore, came to the conclusion that the crust of the earth must be at least

[30] See *Book I, Chapter V, Article 216.*

from 800 to 1,000 miles in thickness; and Sir William Thomson[31] believes that 2,000 or 2,500 miles would not be an over-estimate of the depth of solid matter absolutely necessary to meet the observed facts. But Mr. Hopkins also suggested that if our globe, in the beginning, was condensed from a nebulous vapour into an incandescent fluid, solidification would begin at the centre and advance towards the surface; and that when the remaining liquid matter became of no great thickness, the surface also would begin to solidify by radiation into space: after which time the further solidification would proceed simultaneously from the outside inwards, and from the inside outwards. The result of this would be that in time the whole would be solidified, or that some portions or pockets of liquefied matter, as Mr. Scrope[32] calls them, only would remain here and there, wherever more intense heat was generated.

The rigidity of the planet as a whole is proved, according to Sir W. Thomson, by the tides; for if the solid parts of the earth had so little rigidity as to yield like a fluid, there would be no tides at all; and this would be equally the case if a thin solid crust rested on a fluid beneath. Therefore he comes to the conclusion that the earth's upper crust is probably nearly as rigid as glass, and that *the earth as a whole must be far more rigid than glass*, and probably even more rigid than steel.[33] This conclusion is, however, still questioned by some physicists. The lunar inequalities of themselves show that the strata of the earth increase in density from the surface to the centre, and the enormous pressure of the superincumbent mass is a sufficient cause for this phenomenon. Professor Leslie[34] calculated that air compressed into the fiftieth part of its volume has its elasticity fifty times augmented. If it continues to contract at that rate, it would, from its own incumbent weight, acquire the density of water at the depth of thirty-four miles. But water itself would have its density doubled at the depth of ninety-three miles, and would even attain the density of quicksilver at a depth of 262 miles. Descending therefore towards the centre through nearly 4,000 miles, the condensation of ordinary substances would surpass the utmost powers of conception. Dr. Young[35] computed that steel would be compressed into one-fourth and stone into one-eighth of its bulk at the earth's centre. We are yet ignorant of the laws of compression of solid bodies beyond a certain limit. But while astronomical considerations contradict the assumption of a fluid nucleus covered only by a thin shell, the opposite conclusion of a continuous solid nucleus of such extreme density as would result from the above experiments is rendered impossible by the fact that the mean density of the earth is only about six times that of water, or a little more than twice that of many rocks near the surface. On the whole, therefore, it would seem that our planet must have a cavernous structure of great rigidity, the extreme density of the central parts being to a considerable extent neutralized by expansion due to great heat.

---

[31] Kelvin (of Largs), William Thomson, Baron (1824-1907), Scottish mathematician and physicist, born in Belfast, N. Ireland. Known for his work in thermodynamics, energy conservation and hydrodynamics. The unit of absolute temperature is named after him. Thomson laid foundations for modern physics in his development of the basis of electromagnetism. He was an early advocate of the idea that all forces in nature would ultimately be related in one unified theory. Thomson later became James Clerk Maxwell's mentor. His many other interests included questions on the shape of the earth.

[32] Scrope, George Julius Poulett, (1797-1776), born London England, geologist whose volcanic theories helped undermine the Neptunist theory: that the world's oldest rocks were sedimentary in origin. His treatise *Considerations on Volcanoes* was published in 1825.

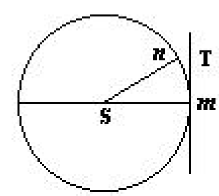
[33] Sir W. Thomson, *Phil. Trans.*, vol. cliii. 1863. (Somerville's note.)

[34] Leslie, John, Sir, (1766-1832), mathematician and physicist, born in Largo, Scotland. Leslie was Chair of mathematics at Edinburgh (1805) and later Physics (1819). Known primarily for his studies on heat, Leslie also invented a differential thermometer, a hygrometer, and a photometer. Leslie published 10 books including *The Philosophy of Arithmetic*.

[35] See note 35, *Preliminary Dissertation.*

The Sun, Moon, Planets and Satellites

The masses of such planets as have no satellites are known by comparing the inequalities they produce in the motions of the earth and of each other, determined theoretically, with the same inequalities given by observation; for the disturbing cause must necessarily be proportional to the effect it produces. The masses of the satellites themselves may also be compared with that of the sun by their perturbations. Thus, it is found, from the comparison of a vast number of observations with Laplace's theory of Jupiter's satellites, that the mass of the sun is more than 60,000,000 times greater than the least of these moons. But, as the quantities of matter in any two primary planets are directly as the cubes of the mean distances at which their satellites revolve, and inversely as the squares of their periodic times,[36] the mass of the sun and of any planets which have satellites may be compared with the mass of the earth.[37] In this manner it is computed that the mass of the sun is 315,000 times that of the earth; whence the great perturbations of the moon, and the rapid motion of the perigee and nodes of her orbit. Even Jupiter, the largest of the planets, has been found by Professor Airy[38] to be 1046.77 times less than the sun; and, indeed, the mass of the whole Jovial system is not more than the 1054.4th part of that of the sun. So that the mass of the satellites bears a very small proportion to that of their primary. The mass



[36] *Inversely &c.* The quantities of matter in any two primary planets are greater in proportion as the cubes of the numbers representing the mean distances of their satellites are greater, and also in proportion as the squares of their periodic times are less. (Somerville's note.)

[37] As hardly anything appears more impossible than that man should have been able to weigh the sun as it were in scales and the earth in a balance, the method of doing so may have some interest. The attraction of the sun is to the attraction of the earth as the quantity of matter in the sun to the quantity of matter in the earth; and, as the force of this reciprocal attraction is measured by its effects, the space the earth would fall through in a second by the sun's attraction is to the space which the sun would fall through by the earth's attraction as the mass of the sun to the mass of the earth. Hence, as many times as the fall of the earth to the sun in a second exceeds the fall of the sun to the earth in the same time, so many times does the mass of the sun exceed the mass of the earth. Thus the weight of the sun will be known if the length of these two spaces can be found in miles or parts of a mile. Nothing can be easier. A heavy body falls through 16.0697 feet in a second at the surface of the earth by the earth's attraction; and, as the force of gravity is inversely as the square of the distance, it is clear that 16.0697 feet are to the space a body would fall through at the distance of the sun by the earth's attraction, as the square of the distance of the sun from the earth to the square of the distance of the centre of the earth from its surface; that is, as the square of 91,600,000 miles to the square of 4000 miles. And thus, by a simple question in proportion the space which the sun would fall through in a second by the attraction of the earth may be found in parts of a mile. The space the earth would fall through in a second, by the attraction of the sun, must now be found in miles also. Suppose *mn*, in the figure above, to be the arc which the earth describes round the sun in S, by the joint action of the sun and the velocity of the planet in a second of time. By the planet's velocity alone the earth would move from *m* to T in a second, and by the sun's attraction alone it would fall through T*n* in the same time. Hence, the length of T*n*, in miles, is the space the earth would fall through in a second by the sun's attraction. Now, as the earth's orbit is very nearly a circle, if 360 degrees be divided by the number of seconds in a sidereal year of 366.25 days, it will give *mn*, the arc which the earth moves through in a second, and then the tables will give the length of the line ST in numbers corresponding to that angle; but, as the radius S*n* is assumed to be unity in the tables, if 1 be subtracted from the number representing ST, the length of T*n* will be obtained; and when multiplied by 91,600,000, to reduce it to miles, the space which the earth falls through, will be obtained in miles. By this simple process it is found that, if the sun were placed in one scale of a balance, it would require 315,000 earths to form a counterpoise. (Somerville's note.)

[38] Airy, George Biddell, Sir, (1801-1892), astronomer, born in Alnwick, England. Airy was professor of mathematics at Cambridge. He calculated the mass of the earth from gravity measurements.

of the moon is determined from several sources–from her action on the terrestrial equator, which occasions the nutation in the axis of rotation; from her horizontal parallax; from an inequality she produces in the sun's longitude; and from her action on the tides. The three first quantities, computed from theory and compared with their observed values, give her mass respectively equal to the $\frac{1}{71}$, $\frac{1}{74.2}$, and $\frac{1}{69.2}$ part of that of the earth, which do not differ much from each other. Dr. Brinkley[39] has found it to be $\frac{1}{80}$ from the constant of lunar nutation: but, from the moon's action in raising the tides, her mass appears to be about the $\frac{1}{75}$ part of that of the earth–a value that cannot differ much from the truth.

The apparent diameters of the sun, moon, and planets are determined by measurement; therefore their real diameters may be compared with that of the earth; for the real diameter of the planet is to the real diameter of the earth, or 7899 miles, as the apparent diameter of the planet to the apparent diameter of the earth as seen from the planet, that is, to twice the parallax of the planet. According to Bessel,[40] the mean apparent diameter of the sun is $1923''.64$, and with the solar parallax $8''.9$, it will be found that the diameter of the sun is about 852,900 miles. Therefore, if the centre of the sun were to coincide with the centre of the earth, his volume would not only include the orbit of the moon, but would extend nearly as far again; for the moon's mean distance from the earth is about sixty times the earth's equatorial radius, or 238,793 miles: so that twice the distance of the moon is 477,586 miles, which does not greatly exceed the solar radius. The diameter of the moon is only 2164 miles; and Jupiter's diameter of 84,800 miles is very much less than that of the sun; the diameter of Ceres, the largest of the minor planets, is only 196 miles, and that of Pallas does not much exceed 171 miles, so that an inhabitant of that planet, in one of our steam carriages, might go round these worlds in a few hours, and the whole of the 158 telescopic planets[41] are so small, that their united mass is almost inappreciable in affecting the movements of the heavenly bodies.

The densities of bodies are proportional to their masses, divided by their volumes. Hence, if the sun and planets be assumed to be spheres, their volumes will be as the cubes of their diameters. Now, the apparent diameters of the sun and earth, at their mean distance, are $1923''.6$ and $17''.1552$, and the mass of the earth is the 315,000th part of that of the sun taken as the unit. It follows, therefore, that the earth is four times as dense as the sun. But the sun is so large that his attractive force is 27.9 times as great as that of the earth. Consequently, if he were habitable by human beings, they would be unable to move, since their weight would be nearly twenty-eight times as great as it is here. A man of moderate size would weigh about two tons at the surface of the sun; whereas at the surface of some of the new planets he would be so light that it would be impossible to stand steady, since he would only weigh a few pounds. The mean density of the earth has been determined by the following method. Since a comparison of the action of two planets upon a third gives the ratio of the masses of these two planets, it is clear that, if we can compare the effect of the whole earth with the effect of any part of it, a comparison may be instituted between the mass of the whole earth and the mass of that part of it. Now a leaden ball was weighed against the earth by comparing the effects of each upon a pendulum; the nearness of the smaller mass making it produce a sensible effect as compared with that of the larger: for by the laws of attraction the whole earth must be considered as collected in its centre. By this

[39] Brinkley, John, *Elements of Astronomy*, Dublin, 1813.
[40] See note 37, *Bk. II, Chap. XIV.*
[41] See note 9, *Preliminary Dissertation.*

method it has been found that the mean density of the earth is 5.660 times greater than that of water at the temperature of 62° of the Fahrenheit's thermometer. The late Mr. Baily,[42] whose accuracy as an experimental philosopher is acknowledged, was unremittingly occupied nearly four years in accomplishing this very important object. In order to ascertain the mean density of the earth still more perfectly, Sir G. B. Airy made a series of experiments to compare the simultaneous oscillations of two pendulums, one at the bottom of the Harton coal-pit, 1260 feet deep, in Northumberland, and the other on the surface of the earth immediately above it. The oscillations of the pendulums were compared with an astronomical clock at each station, and the time was instantaneously transmitted from one to the other by a telegraphic wire. The oscillations were observed for more than 100 hours continuously, when it was found that the lower pendulum made $2\frac{1}{2}$ oscillations more in 24 hours than the upper one. The experiment was repeated for the same length of time with the same result; but on this occasion the upper pendulum was taken to the bottom of the mine and the lower brought to the surface. From the difference between the oscillations at the two stations it appears that gravitation at the bottom of the mine exceeds that at the surface by the $\frac{1}{19190}$ part, and that the mean density of the earth is 6.565, which is greater than that obtained by Mr. Baily by 0.89, but there are many reasons why this result cannot be so exact as could be wished. While employed on the trigonometrical survey of Scotland, Colonel James determined the mean density of the earth to be 5.316, from a deviation of the plumb-line amounting to $2''$, caused by the attraction of Arthur's Seat[43] and the heights east of Edinburgh: it agrees more nearly with the density found by Mr. Baily than with that deduced from Sir G. B. Airy's experiments, and upon the whole 5.6 seems to be the nearest estimate we can assume for the earth's density. All the planets and satellites, except perhaps Mercury, appear to be of less density than the earth.

Before entering on the theory of rotation, it may not be foreign to the subject to give some idea of the methods of computing the places of the planets, and of forming astronomical tables. Astronomy is now divided into the three distinct departments of theory, observation, and computation. Since the problem of the three bodies can only be solved by approximation, the analytical astronomer determines the position of a planet in space by a series of corrections. Its place in its circular orbit is first found, then the addition or subtraction of the equation of the centre (see art. 382, *Bk. II, Chap. IV*) to or from its mean place gives its position in the ellipse. This again is corrected by the application of the principal periodic inequalities. But, as these are determined for some particular position of the three bodies, they require to be corrected to suit other relative positions. This process is continued till the corrections become less than the errors of observation, when it is obviously unnecessary to carry the approximation further. The true latitude and distance of the planet from the sun are obtained by methods similar to those employed for the longitude.

As the earth revolves equably about its axis in 24 hours, at the rate of 15° in an hour, time becomes a measure of angular motion, and the principal element in astronomy, where the object is to determine the exact state of the heavens and the successive changes it undergoes in all ages, past, present, and to come. Now, the longitude, latitude, and distance of a planet from the sun are given in terms of the time, by general analytical formulae. These formulae will consequently give the exact place of the body in the heavens, for any time assumed at pleasure, provided they can be reduced to numbers. But before the calculator begins his task the observer must furnish

---

[42] See note 11, *Bk. I, Foreword.*

[43] A well known ridge near Edinburgh.

the necessary data, which are, obviously, the forms of the orbits, and their positions with regard to the plane of the ecliptic. It is therefore necessary to determine by observation, for each planet, the length of the major axis of its orbit, the eccentricity, the inclination of the orbit to the plane of the ecliptic, the longitudes of its perihelion and ascending node at a given time, the periodic time of the planet, and its longitude at any instant arbitrarily assumed, as an origin from whence all its subsequent and antecedent longitudes are estimated. Each of these quantities is determined from that position of the planet on which it has most influence. For example, the sum of the greatest and least distances of the planet from the sun is equal to the major axis of the orbit, and their difference is equal to twice the eccentricity. The longitude of the planet, when at its least distance from the sun, is the same with the longitude of the perihelion: the greatest latitude of the planet is equal to the inclination of the orbit: the longitude of the planet, when in the plane of the ecliptic in passing towards the north, is the longitude of the ascending node, and the periodic time is the interval between two consecutive passages of the planet through the same node, a small correction being made for the precession of the node during the revolution of the planet. Notwithstanding the excellence of instruments and the accuracy of modern observers, unavoidable errors of observation can only be compensated by finding the value of each element from the mean of a thousand, or even many thousands of observations. For as it is probable that the errors are not all in one direction, but that some are in excess and others in defect, they will compensate each other when combined.

However, the values of the elements determined separately can only be regarded as approximate, because they are so connected that the estimation of any one independently will induce errors in the others. The eccentricity depends upon the longitude of the perihelion, the mean motion depends upon the major axis, the longitude of the node upon the inclination of the orbit, and *vice versâ*. Consequently, the place of a planet computed with the approximate data will differ from its observed place. Then the difficulty is to ascertain what elements are most in fault, since the difference in question is the error of all; that is obviated by finding the errors of some thousands of observations, and combining them, so as to correct the elements simultaneously, and to make the sum of the squares of the errors a minimum with regard to each element. The method of accomplishing this depends upon the Theory of Probabilities; a subject fertile in most important results in the various departments of science and of civil life, and quite indispensable in the determination of astronomical data. A series of observations continued for some years will give approximate values of the secular and periodic inequalities, which must be corrected from time to time, till theory and observation agree. And these again will give values of the masses of the bodies forming the solar system, which are important data in computing their motions. The periodic inequalities derived from a great number of observations are employed for the determination of the values of the masses till such time as the secular inequalities shall be perfectly known, which will then give them with all the necessary precision. When all these quantities are determined in numbers, the longitude, latitude, and distance of the planet from the sun are computed for stated intervals, and formed into tables, arranged according to the time estimated from a given epoch, so that the place of the body may be determined from them by inspection alone, at any instant for perhaps a thousand years before and after that epoch. By this tedious process, tables have been computed for all the great planets, and several of the small, besides the moon and the satellites of Jupiter. In the present state of astronomy the masses and elements of the orbits are pretty well known, so that the tables only require to be corrected from time to time as observations become more accurate. Those containing the motions of Jupiter, Saturn, and Uranus have already been three times constructed within the last fifty years, and the tables of Jupiter and Saturn agree almost perfectly with modern observation. The following prediction will be found in the sixth edition of *Connexion of the Physical Sciences*,[44] published in the year 1842: '*Those of Uranus, however, are already defective, probably because the discovery of that planet in 1781 is too recent to admit of much precision in the determination of its motions, or that possibly it may be subject to disturbances from some unseen planet revolving about the sun beyond the present boundaries of our system. If, after a lapse of years, the tables formed from a combination of numerous observations should be still inadequate to represent the motions of Uranus, the discrepancies may reveal the existence, nay, even the mass and orbit, of a body placed for ever beyond the sphere of vision.*'

That prediction has been fulfilled since the seventh edition of that book was published.[45] Not only the existence of Neptune, revolving at the distance of two thousand seven hundred millions of miles from the sun, has been discovered from his disturbing action on Uranus, but his mass, the form and position of his orbit in space, and his periodic time had been determined before the planet had been seen, and the planet itself was discovered in the very point of the heavens which had been assigned to it. It had been noticed for years that the perturbation of Uranus had increased in an unaccountable manner.[46] After the disturbing action of all the known planets had been determined, it was found that, between the years 1833 and 1837, the observed and computed distance of Uranus from the sun differed by 24,000 miles, which is about the mean distance of the moon from the earth, while, in 1841, the error in the geocentric longitude of the planet amounted to $96''$. These discrepancies were therefore attributed to the attraction of some unseen and unknown planet, consequently they gave rise to a case altogether unprecedented in the history of astronomy. Heretofore it was required to determine the disturbing action of one known planet upon another. Whereas the inverse problem had now to be solved, in which it was required to find the place of an unknown body in the heavens, at a given time, together with its mass, and the form and position of its orbit, from the disturbance it produced on the motions of another. The difficulty was extreme, because all the elements of the orbit of Uranus were erroneous from the action of Neptune, and those of Neptune's orbit were unknown. In this dilemma it was necessary to form some hypothesis with regard to the unknown planet; it was therefore assumed, according to Bode's empirical law on the mean distances of the planets, that it was revolving at twice the distance of Uranus from the sun. In fact, the periodic time of Uranus is about 84 years, and, as the discrepancies in his motions increased slowly and regularly it was evident that it would require a planet with a much longer periodic time to produce them–moreover, it was clear that the new planet must be exterior to Uranus, otherwise it would have disturbed the motions of Saturn.

Another circumstance tended to lessen the difficulty; the latitude of Uranus was not much affected, therefore it was concluded that the inclination of the orbit of the unknown body must be very small, and, as that of the orbit of Uranus is only $46' 28''.4$, both planets were assumed to be moving in the plane of the ecliptic, and thus the elements of the orbit of the unknown planet were reduced from six to four. Having thus assumed that the unknown body was revolving in a circle in the plane of the ecliptic, the analytical expression of its action on the motion of Uranus, when

---

[44] Somerville, Mary, *On the Connexion of the Physical Sciences*, 6th edition, London : John Murray, 1842

[45] Neptune was discovered in the year 1846. (Somerville note.)

[46] The true longitude of Uranus was in advance of the tables previous to 1795, and continued to advance till 1822, after which it diminished rapidly till 1830-1, when the observed and calculated longitudes agreed, but then the planet fell behind the calculated place so rapidly that it was clear the tables could no longer represent its motion. (Somerville's note.)

in numerous points of its orbit, was compared with the observed longitude of Uranus, through a regular series of years, by means of which the faulty elements of the orbit of Uranus were eliminated, or got rid of, and there only remained a relation between the mass of the new planet and three of the elements of its orbit; and it then was necessary to assume such a value for two of them as would suit the rest. That was accomplished so dexterously, that the perturbations of Uranus were perfectly conformable to the motions of Neptune, moving in the orbit thus found, and the place of the new planet exactly agreed with observation. Subsequently its orbit and motions have been determined more accurately.

The honour of this admirable effort of genius is shared by Mr. Adams and M. Leverrier,[47] who, independently of each other, arrived at these wonderful results. Mr. Adams had determined the mass and apparent diameter of Neptune, with all the circumstances of its motion, eight months before M. Leverrier had terminated his results, and had also pointed out the exact spot where the planet would be found; but the English observers neglected to look for it till M. Leverrier made known his researches, and communicated its position to Dr. Galle,[48] at Berlin, who found it the very first night he looked for it, and then it was evident that it would have been seen in the place Mr. Adams had assigned to it eight months before had it been looked for. So closely did the results of these two great mathematicians agree.

Neptune has a diameter of 37,314 miles,[49] consequently he is nearly 200 times larger than the earth, and may be seen with a telescope of moderate power. His motion is retrograde at present, and six times slower than that of the earth. At so great a distance from the sun it can only have the $\frac{1}{1300}$ part of the light and heat the earth receives; but having a satellite,[50] the deficiency of light may in some measure be supplied.

The prediction may now be transferred from Uranus to Neptune, whose perturbations may reveal the existence of a planet still further removed, which may for ever remain beyond the reach of telescopic vision—yet its mass, the form and position of its orbit, and all the circumstances of its motion may become known, and the limits of the solar system may still be extended hundreds of millions of miles.[51]

The mean distance of Neptune from the sun has subsequently proved to be only 2746 millions of miles, and the period of his revolution 164 years, so that Baron Bode's law, of the interval between the orbits of any two planets being twice as great as the inferior interval and half of the superior, fails in the case of Neptune, though it was useful on the first approximation to his motions; and since Bode's time it has led to the discovery of 150 telescopic planets revolving between the orbits of Mars and Jupiter.

The tables of Mars, Venus, and even those of the sun, have been greatly improved, and, together with those of Jupiter, Saturn, and Uranus, form the basis of a grand work by M. Leverrier, which is only now (1876) approaching completion, after having occupied that eminent

---

[47] See note 28, *Bk. II, Foreword.*

[48] Galle, Johann Gottfried, (1812-1910), astronomer who observed the planet Neptune at the Berlin Observatory on September 23, 1846. Galle looked for and located the planet a few days after a request from the French astronomer Leverrier who had independently calculated the location of the planet eight months after J. C. Adams had made a similar request in September 1845 to the Cambridge Observatory. (see also *Introduction to the Second Edition*; notes 28 & 39, *Bk. II, Foreword*; and note 38, *Bk. II, Chap. XIV*.)

[49] The modern value for Neptune's equatorial diameter is 30,777 miles (49,528 km). The polar diameter is 30,200 (48,600 km).

[50] The largest of Neptune's eight known satellites, *Triton*, was discovered by William Lassell (1799-1880), a few weeks after the discovery of Neptune in 1846.

[51] See note 23, *Foreword to the Second Edition.*

astronomer during twenty years. We are chiefly indebted to the German astronomers for tables of the four older telescopic planets, Vesta, Juno, Ceres, and Pallas, the others have only been discovered since the year 1845.

The determination of the path of a planet when disturbed by all the others, a problem which has employed the talents of the greatest astronomers, from Newton to the present day is only completely accomplished with regard to the older planets, which revolve in nearly circular orbits, but little inclined to the plane of the ecliptic. When the eccentricity and inclination of the orbits are great, analysis becomes almost impossible, because the series expressing the co-ordinates of the bodies become extremely complicated, and do not converge when applied to comets and the telescopic planets. This difficulty has, at last, been overcome by mathematicians, and many of the secular variations and mutual relations of the orbits of the asteroids or minor planets have been worked out. The problem is, however, one of extreme difficulty, and it must be long before they can all be included together with the larger planets in one comprehensive theory of planetary motion.